

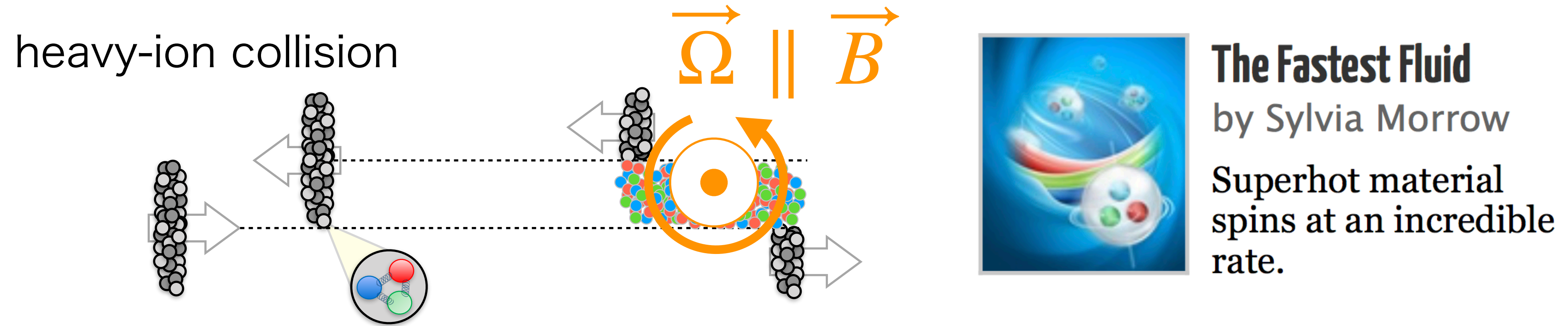
Sign-inversion of magnetovortical charge from gauge invariant thermodynamics

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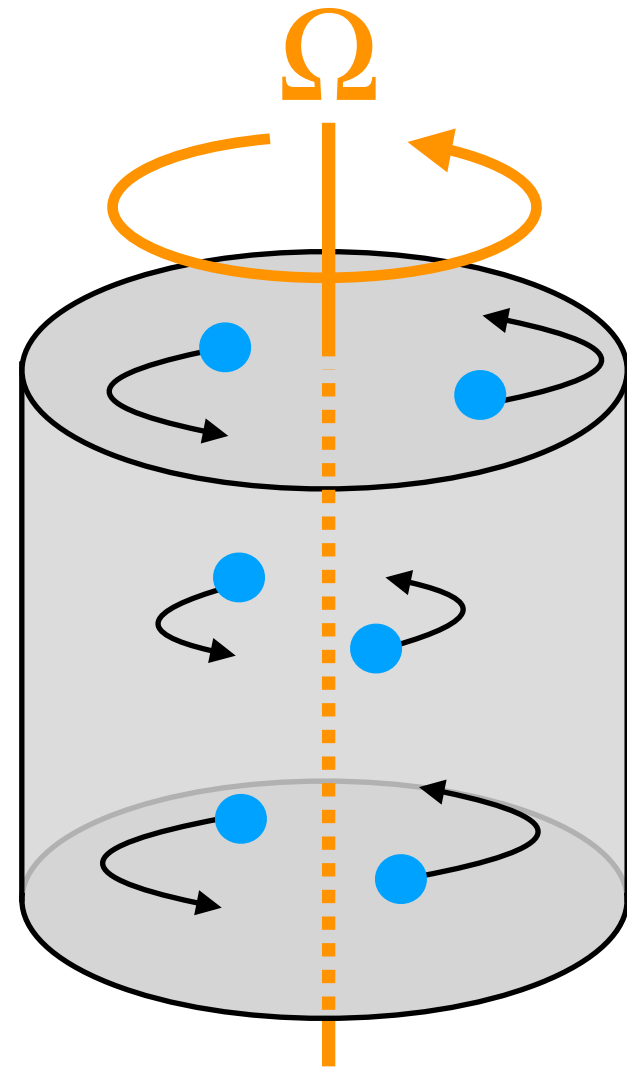
K. Fukushima, K. Hattori and KM, arXiv:2409.***** [hep-ph]

QCD matter under rotation



- ✓ elementary particles affected by $\vec{\Omega}$ (= source of angular momentum)
- ✓ $\vec{\Omega} \parallel \vec{B}$ is more crucial than either $\vec{\Omega}$ or \vec{B}

Early attempt : Thermodynamics



Landau-Lifshitz (1958) Vilenkin (1979)

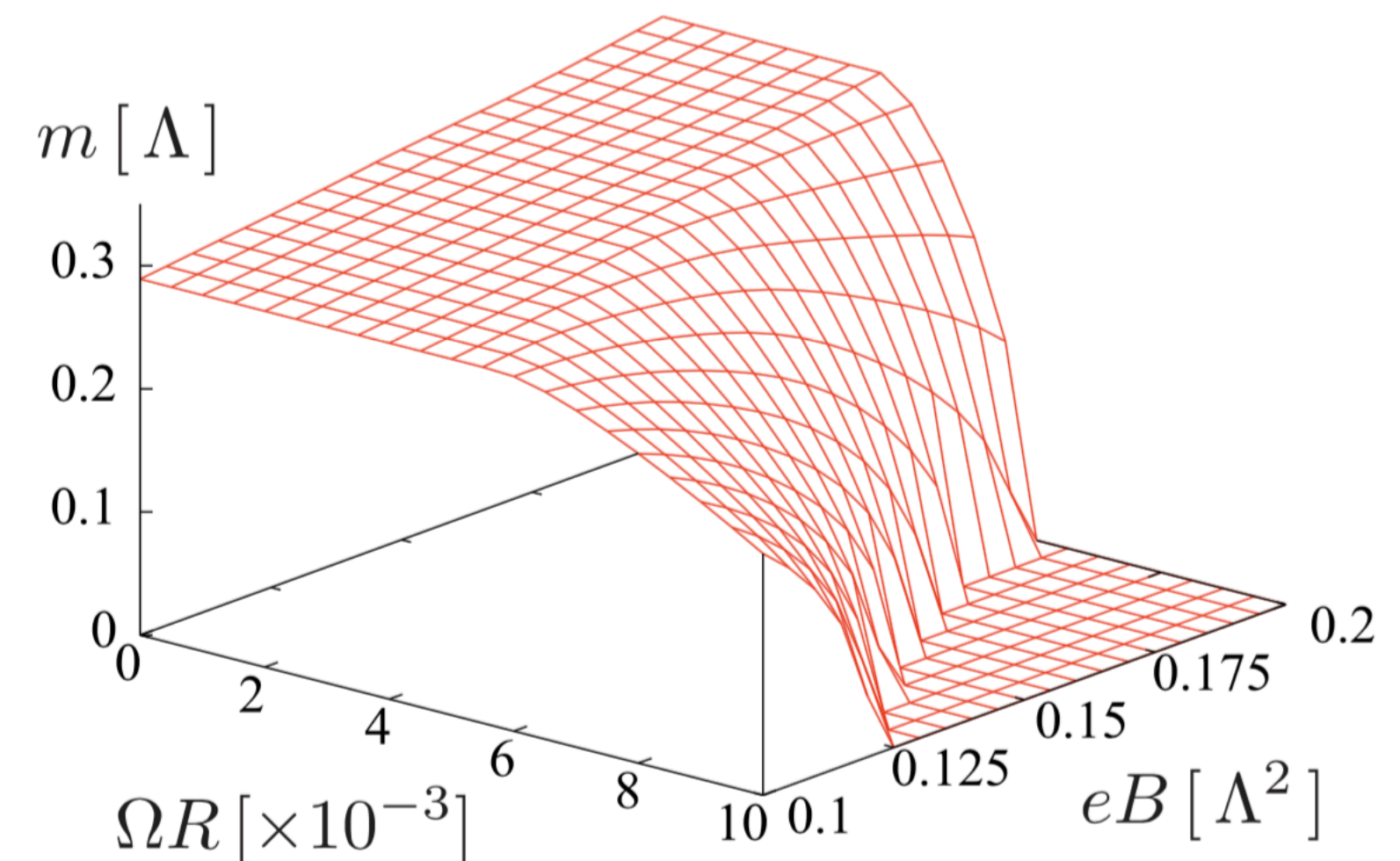
$$Z = \text{tr} \exp[-\beta(H - \Omega \mathcal{J})] \longleftrightarrow H - \mu N$$

Chen-Fukushima-Huang-Mameda (2016)

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega (L + S) \right]$$

↑
purely magnetic part

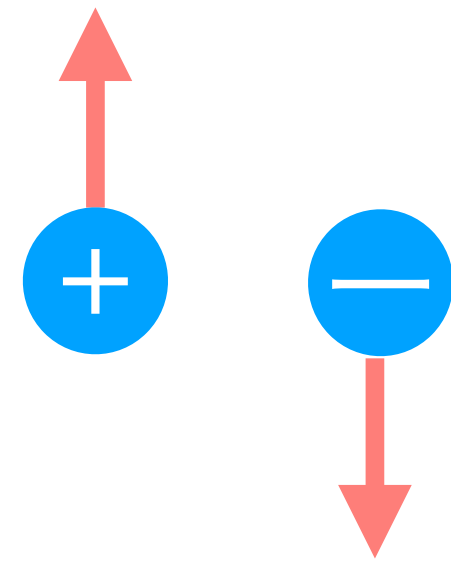
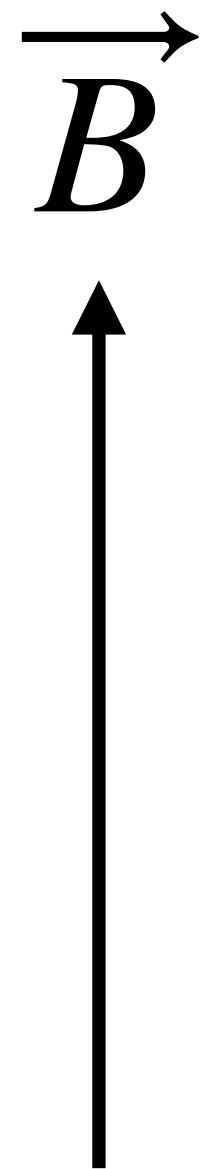
inverse magnetic catalysis



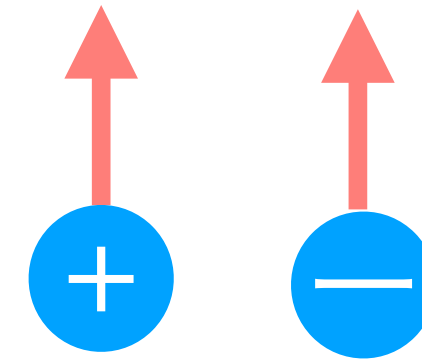
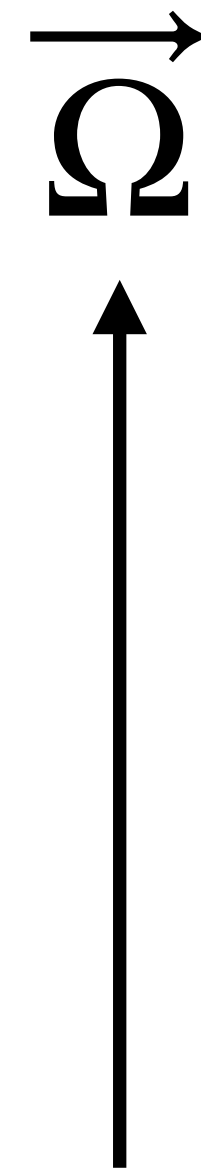
Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$



Zeeman coupling




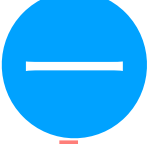
spin-rotation coupling

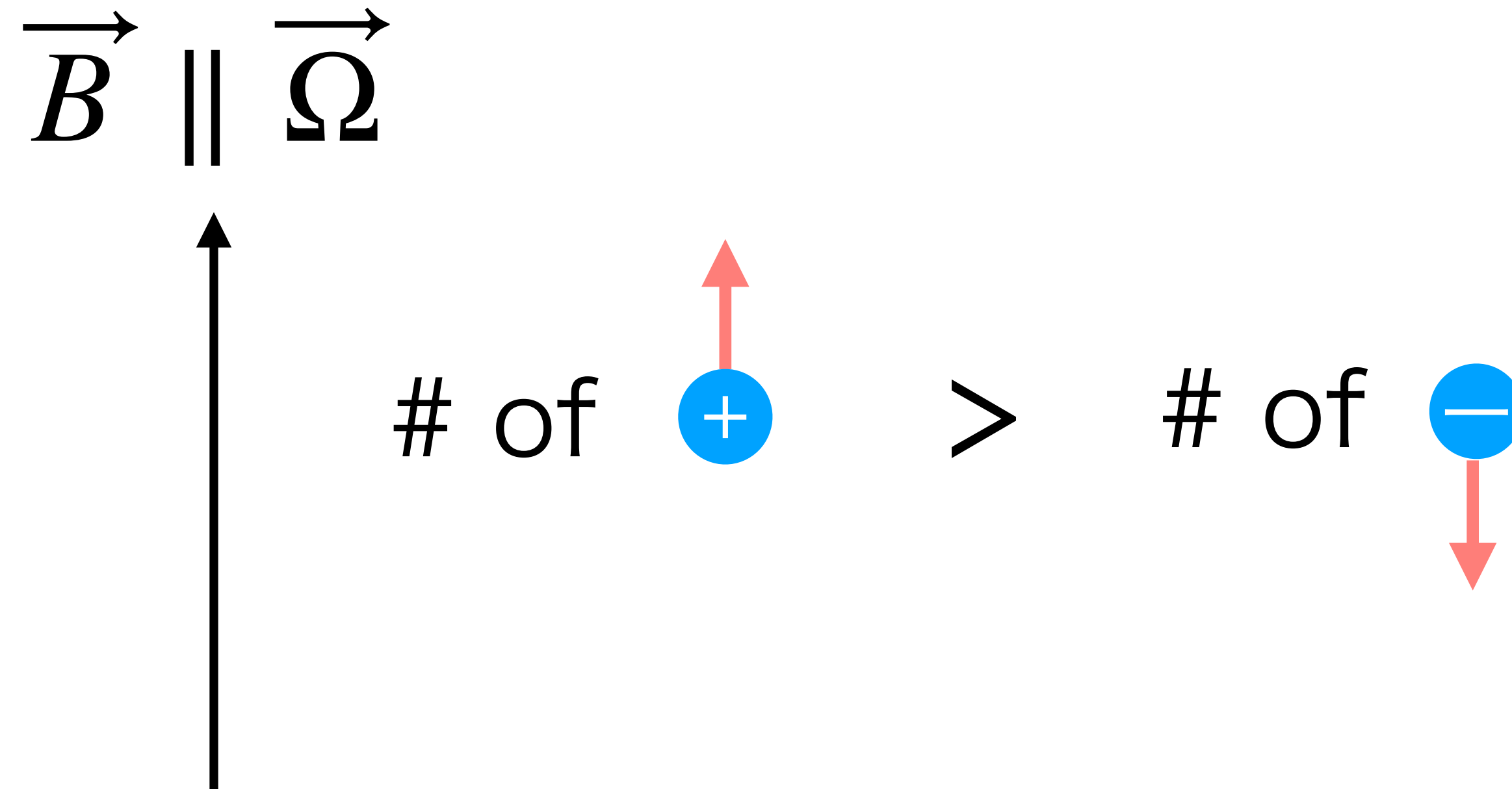
Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

$$\vec{B} \parallel \vec{\Omega}$$

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Puzzle on magneto-vortical charge

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2} \quad \text{Hattori-Yin (2016)}$$

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Chen-Fukushima-Huang-Mameda (2016)

Ebihara-Fukushima-Mameda (2017)

Answer?

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2} \quad \text{Hattori-Yin (2016)}$$

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Final answer

Fukushima-Hattori-Mameda (in prep.)

correct Kubo formula

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

correct free energy

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

I will convince you!

Choice of angular momenta

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega (\mathbf{L} + \mathbf{S}) \right]$$

Chen-Fukushima-Huang-Mameda (2016)

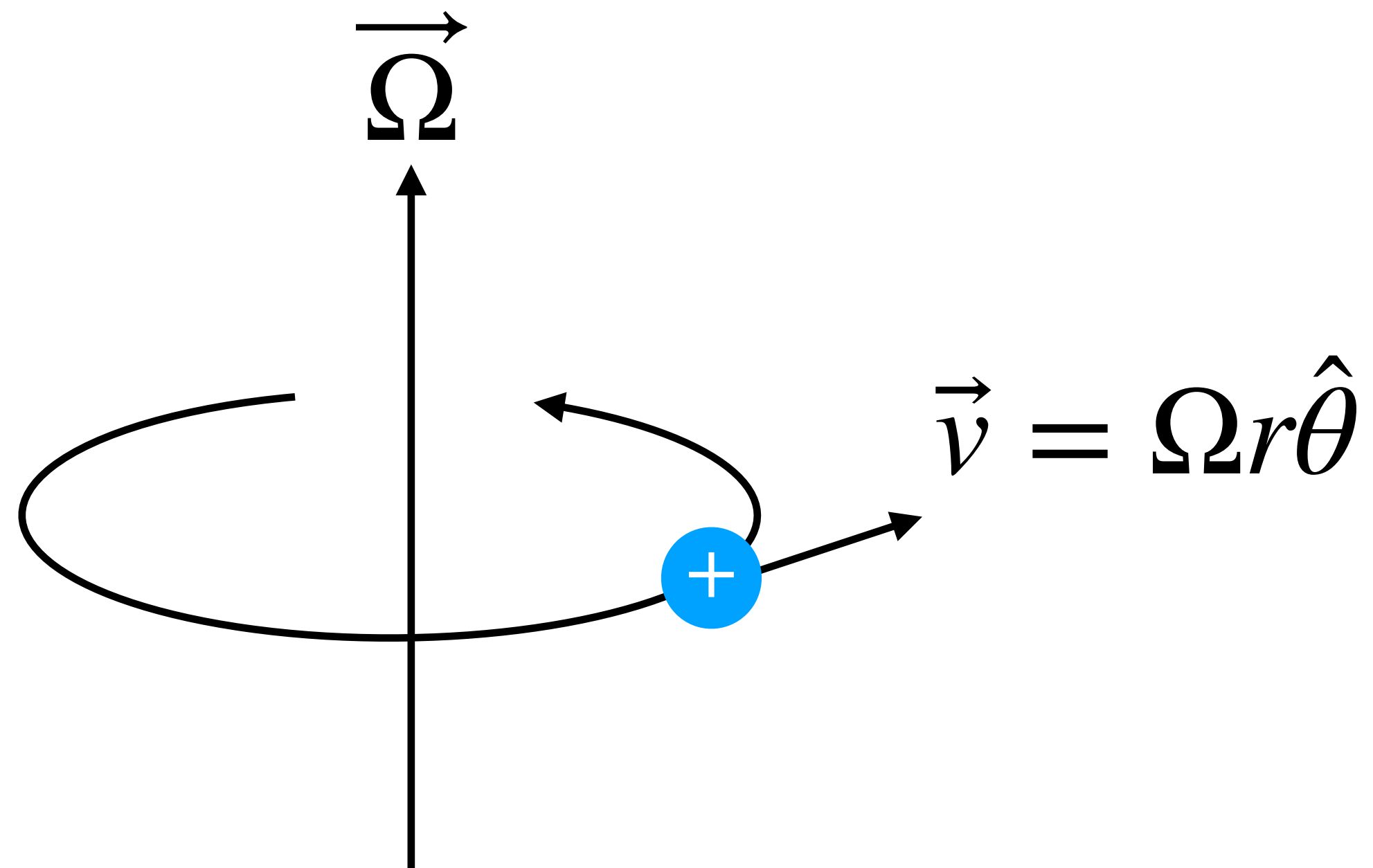
$$L_{\text{can}} = -i(x\partial_y - y\partial_x) \quad \text{conserved AM}$$

Fukushima-Hattori-Mameda (in prep.)

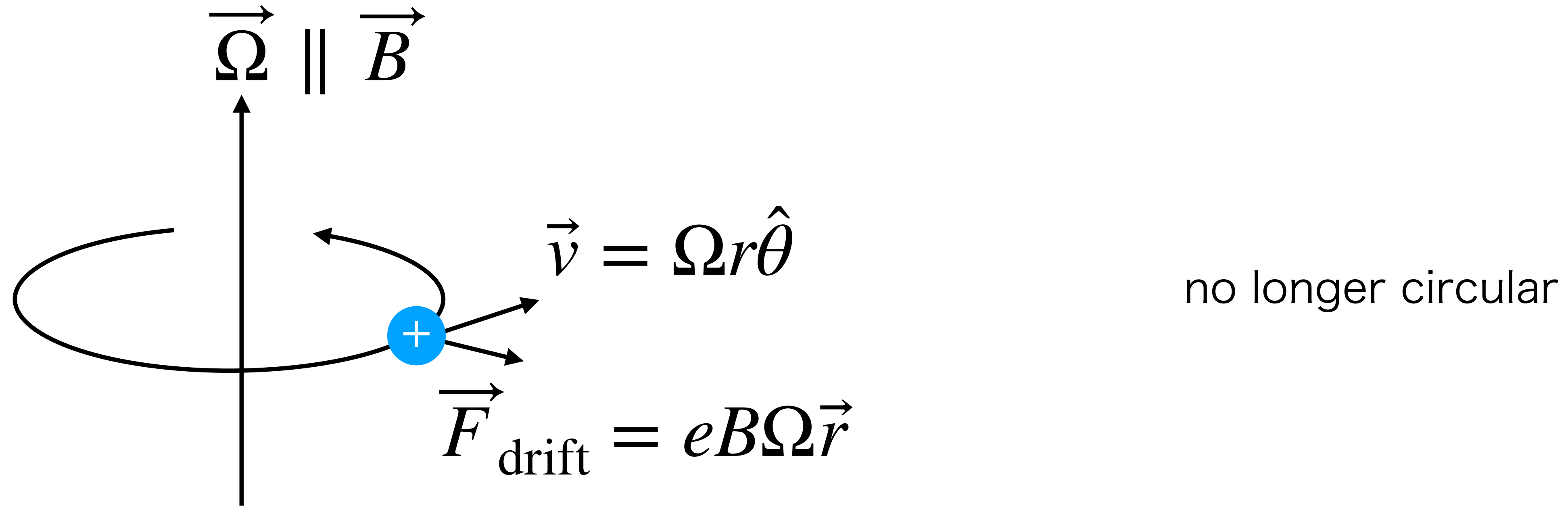
$$L_{\text{kin}} = -i(xD_y - yD_x) \quad \text{gauge invariant AM}$$

cf. Buzzegoli (2020)

Classical interpretation

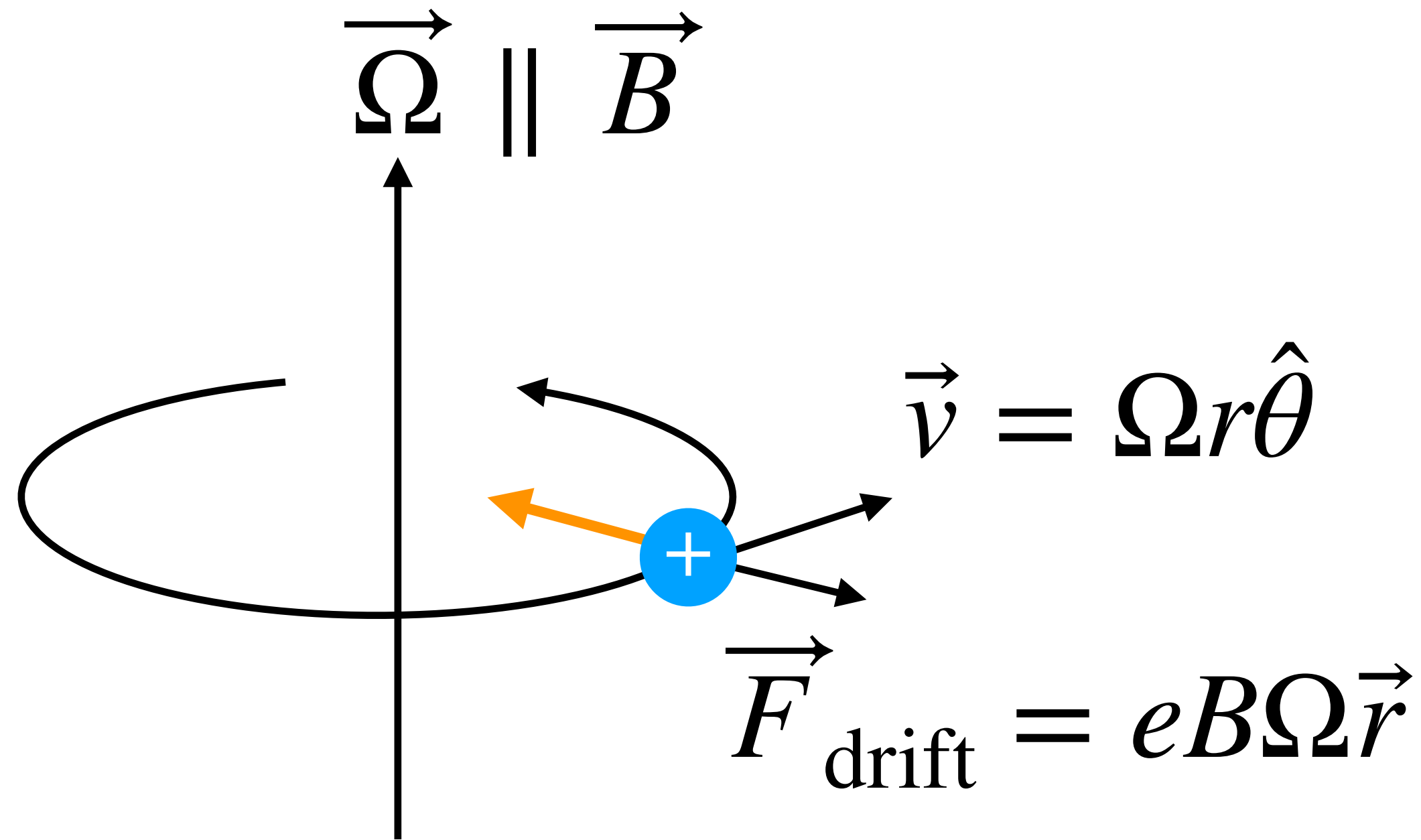


Classical interpretation



$$H - \Omega L_{\text{can}} \quad \text{unstable}$$

Classical interpretation



$$\begin{aligned}
 e\vec{E} &= -eB\Omega\vec{r} \\
 &= -\vec{\nabla} [\Omega(L_{\text{can}} - L_{\text{kin}})] \\
 &\text{for symmetric gauge}
 \end{aligned}$$

$$H + \Omega(L_{\text{can}} - L_{\text{kin}}) - \Omega L_{\text{can}} = H - \Omega L_{\text{kin}} \quad \text{stable}$$

gauge invariance \longleftrightarrow thermodynamic stability

Almost solved?

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega (\mathbf{L} + \mathbf{S}) \right]$$

Chen-Fukushima-Huang-Mameda (2016)

~~$$L_{\text{can}} = -i(x\partial_y - y\partial_x)$$~~

conserved AM

Fukushima-Hattori-Mameda (in prep.)

$$L_{\text{kin}} = -i(xD_y - yD_x)$$

gauge invariant AM

cf. Buzzegoli (2020)

Almost solved?

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega(L + S) \right]$$

How to diagonalize this Dirac operator?

Chen-Fukushima-Huang-Mameda (2016)

~~$$L_{\text{can}} = -i(x\partial_y - y\partial_x)$$~~

conserved AM

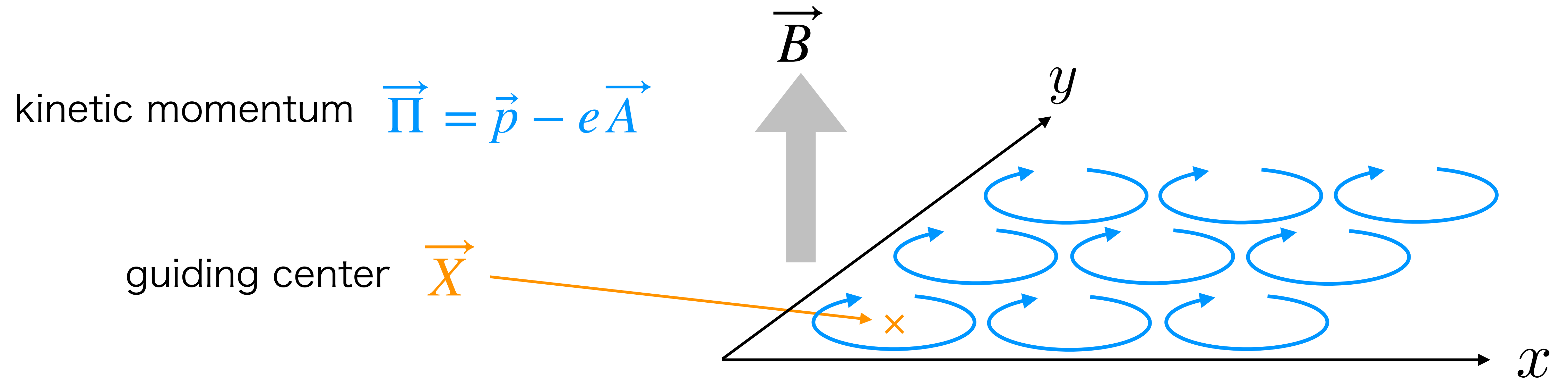
Fukushima-Hattori-Mameda (in prep.)

$$L_{\text{kin}} = -i(xD_y - yD_x)$$

gauge invariant AM

cf. Buzzegoli (2020)

Quantum mechanics



Landau level basis $|n, m\rangle \propto (a^\dagger)^n (b^\dagger)^m |0,0\rangle$

kinetic energy

$$\vec{\Pi}^2 = eB(2a^\dagger a + 1)$$

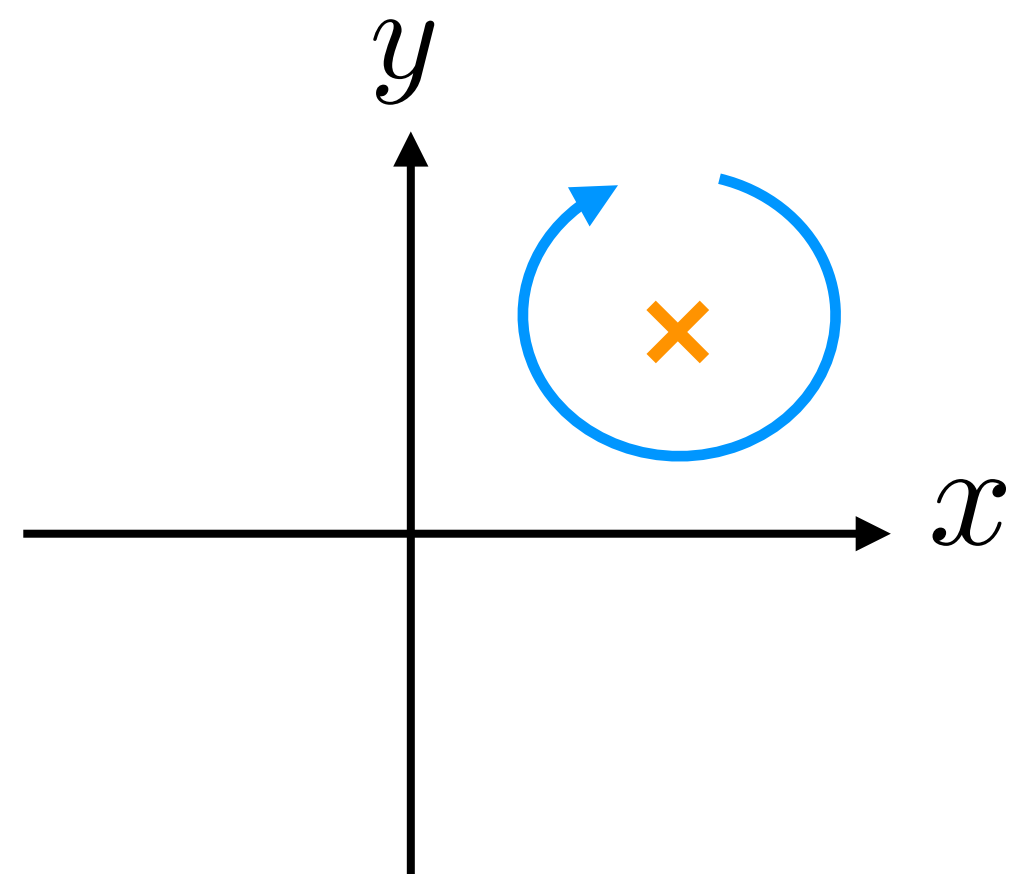
distance from origin

$$\vec{X}^2 = (2b^\dagger b + 1)/eB$$

Angular momenta

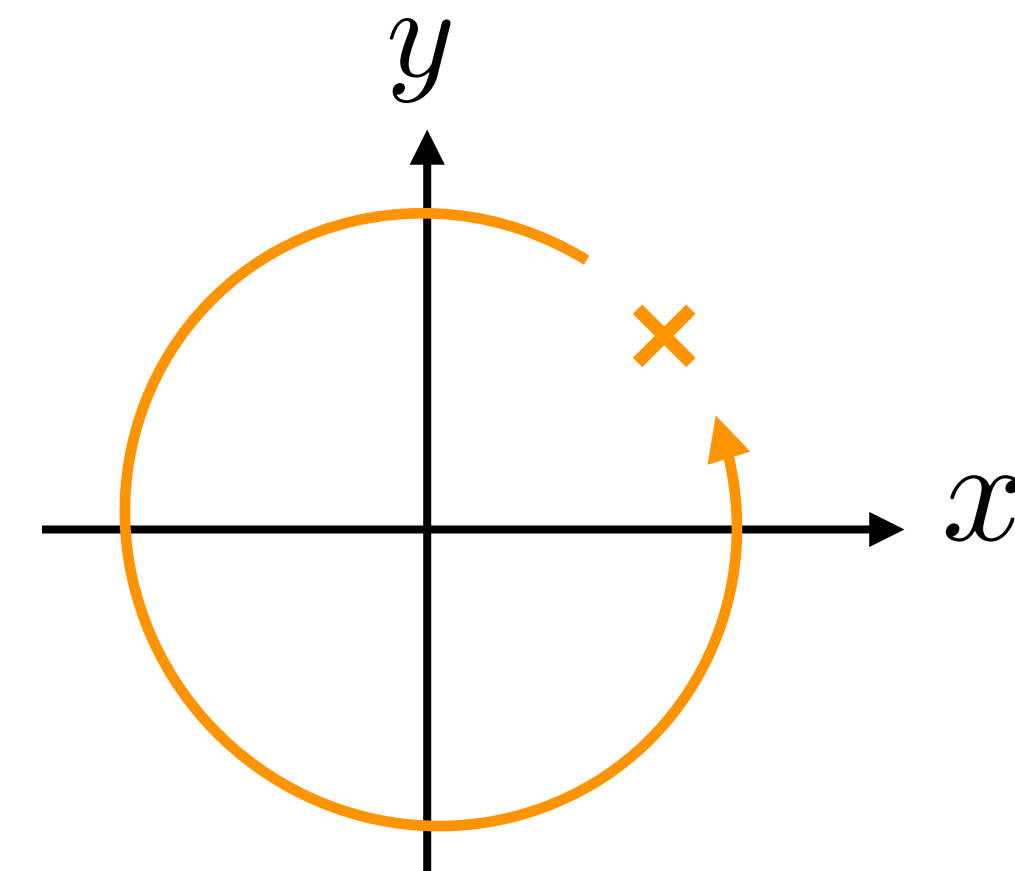
$$L_{\text{kin}} = x\Pi_y - y\Pi_x = \Lambda + \Delta$$

$$\Lambda = (x - X)\Pi_y - (y - Y)\Pi_x \quad \Delta = X\Pi_y - Y\Pi_x$$



cyclotron motion

$$= -(2a^\dagger a + 1)$$



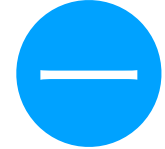
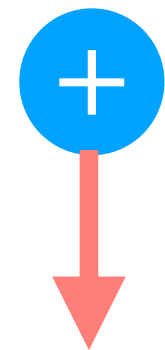
circular motion of guiding center

$$= i(a^\dagger b^\dagger - ab)$$

Lowest Landau Level (LLL)

$$\langle J_{\text{kin}} \rangle_{\text{LLL}} = \langle \underbrace{\Lambda}_{-1} + \underbrace{\Delta}_{0} + \underbrace{S}_{+1/2} \rangle_{\text{LLL}} = -1/2$$

\vec{B}



Zeeman + cyclotron

$\vec{\Omega}$




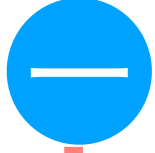
spin-rotation coupling

Lowest Landau Level (LLL)

$$\langle J_{\text{kin}} \rangle_{\text{LLL}} = \langle \Lambda + \Delta + S \rangle_{\text{LLL}} = -1/2$$

$-1 \quad 0 \quad +1/2$

$$\vec{B} \parallel \vec{\Omega}$$

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This suggests

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

LLL Pressure

Fukushima-Hattori-KM (in prep.)

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega (\Lambda + \Delta + \mathcal{S}) \right]$$

in general, not diagonalized analytically

LLL Pressure

Fukushima-Hattori-KM (in prep.)

$$Z = \det \left[\mathcal{D}_B - \gamma^0 \Omega (\Lambda + \Delta + \mathcal{S}) \right]$$

$\nu = -\Omega/2$ (LLL)

$$P_{\text{LLL}} = \frac{eB}{2\pi} \int \frac{dp_z}{2\pi} \left[\ln \left(1 + e^{-\beta(\epsilon - \nu)} \right) + \ln \left(1 + e^{-\beta(\epsilon + \nu)} \right) \right]$$

massless limit

$$\rho = \frac{\partial P_{\text{LLL}}}{\partial \nu} = -\frac{eB\Omega}{4\pi^2} \quad (T\text{-independent})$$

Comparisons

Fukushima-Hattori-Mameda (in prep.)
free energy (LLL)

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

Ebihara-Fukushima-Mameda (2017)
incorrect

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}} + (\underbrace{\text{divergence w.r.t. AM}}_{\text{due to } \vec{F}_{\text{drift}} = eB\Omega\vec{r}})$$

Hattori-Yin (2016)
linear response (LLL)
incorrect

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}} \quad \text{a mistake found}$$

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}} \quad \text{no Landau level formed by weak } B$$

Anomaly-related

$$\rho = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2}$$

for any B only for strong B

spin orbital

Only **the spin part** is due to chiral anomaly?

Hattori-Yin (2016)

YES

This is T -independent

Yang-Yamamoto (2021)

YES

This is derived as a Chern-Simons current

Anomaly-related

charge

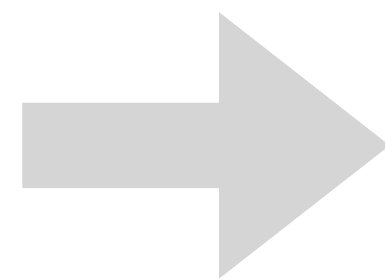
$$\rho = \frac{\partial P_{LLL}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

spin orbital

angular momentum

$$J = \frac{\partial P_{LLL}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{LLL}}{\partial \mu \partial \Omega}$$



same coefficients shared

Anomaly-related

charge

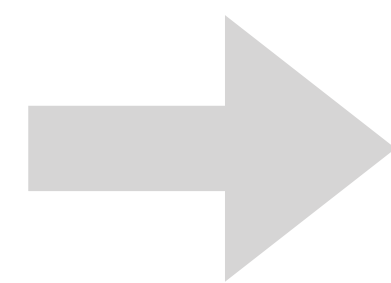
$$\rho = \frac{\partial P_{LLL}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

spin orbital

angular momentum

$$J = \frac{\partial P_{LLL}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$
$$= j_{\text{CSE}}^5 / 2$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{LLL}}{\partial \mu \partial \Omega}$$



Since j_{CSE}^5 is anomaly-related, so is ρ

Moment of inertia of QCD vacuum

$$J = \frac{\partial P_{\text{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

moment of inertia

$$I = \frac{\partial J}{\partial \Omega} = \frac{eB}{8\pi^2} \quad (T\text{-independent})$$

QCD vacuum = color magnetic media [Savvidy \(1977\)](#)

$$I_{\text{QCD vacuum}} = \frac{N_f g \mathcal{B}}{8\pi^2} + I_{\text{gluon}} \stackrel{?}{\neq} 0$$

cf. bag model analysis [Mameda-Takizawa \(2023\)](#)

Summary

- ✓ reformulate gauge-invariant and stable thermodynamics
- ✓ discover cyclotron contribution to magneto-vortical charge
- ✓ The charge is anomaly-unprotected part?
- ✓ kinder to lattice than pure rotation : no boundary effect, homogeneity
- ✓ applicability to various fields

HIC : spin polarization under strong B

neutron stars, electron systems, cold atoms, quantum optics