Strongly interacting QCD matter under extrem conditions

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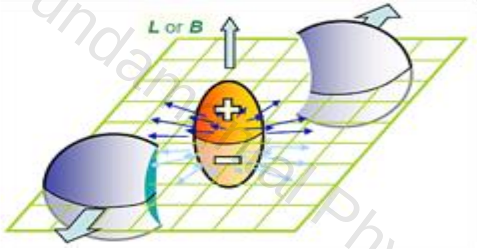
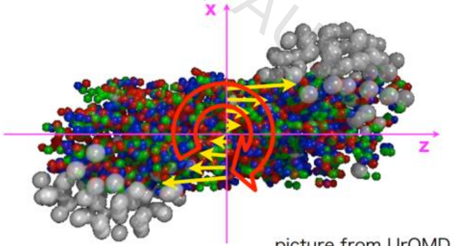
安徽理工大学"基础物理研究中心"成立大会暨学术报告

Outlines

- **Introduction and motivation**
- **Phase structure under rotation and Magnetic field**
- **Transport properties rotating magnetized matter**
- **Summary**

QCD under new extreme conditions



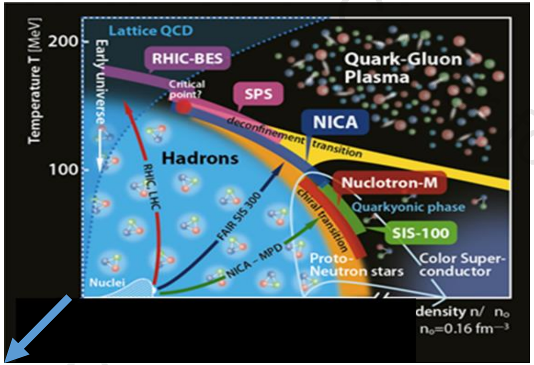


picture from UrQMD

$$E, B \sim \gamma \frac{Z\alpha_{EM}}{R_A^2} \sim 3m_\pi^2$$

$$L \approx \frac{A\sqrt{s_{NN}}}{2} b\hbar \sim 10^6\hbar$$

Explore the new dimensions of the QCD phase diagram



B,
ω

Khazeev, Liao Nature 2021, Becattini- Karpenko etal 2015, 2016; Jiang-Lin-Liao 2016; Deng-XGH-Ma-Zhang 2020; Deng-XGH 2016, Xie-Csernai etal 2014; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017;… …)

New theoretical techniques needed!

Lattice QCD

difficulty with Finite baryon density, Real time dynamics Continuum

Phenom. effective models: (p)NJL、(p)QMC...

Field Theory: HD(T)L , pQCD, xPT

DS Equations

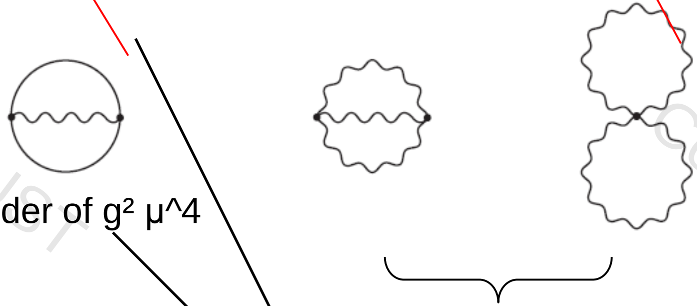
FRGE, Sum rules ....

AdS/CFT, AdS/QCD

CJT effective action of QCD

$$\Gamma[\bar{D},\bar{S}]=\frac{1}{2}\{Tr\ln\bar{D}^{-1}+Tr(D^{-1}\bar{D}-1)-Tr\ln\bar{S}^{-1}-Tr(S^{-1}\bar{S})-2\Gamma_2[\bar{D},\bar{S}]$$

2-loop approximation



Order of g² μ^4

Powers of T

Stationary points

$$\left.\frac{\delta\Gamma}{\delta\bar{\mathcal{D}}}\right|_{\mathcal{D}=\mathcal{D},\mathcal{S}=\mathcal{S}}=0\,,\qquad\left.\frac{\delta\Gamma}{\delta\bar{\mathcal{S}}}\right|_{\mathcal{D}=\mathcal{D},\mathcal{S}=\mathcal{S}}=0$$

$$\Downarrow\qquad\qquad\Downarrow$$

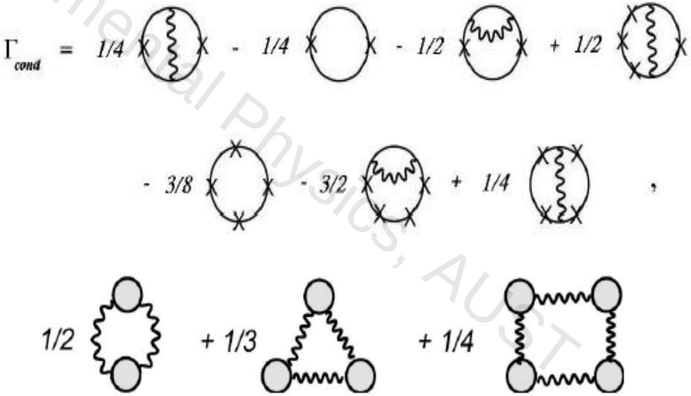
$$\mathcal{D}^{-1}=D^{-1}+\Pi[\mathcal{S}]\qquad\mathcal{S}^{-1}=\mathcal{S}_0^{-1}+\Sigma$$

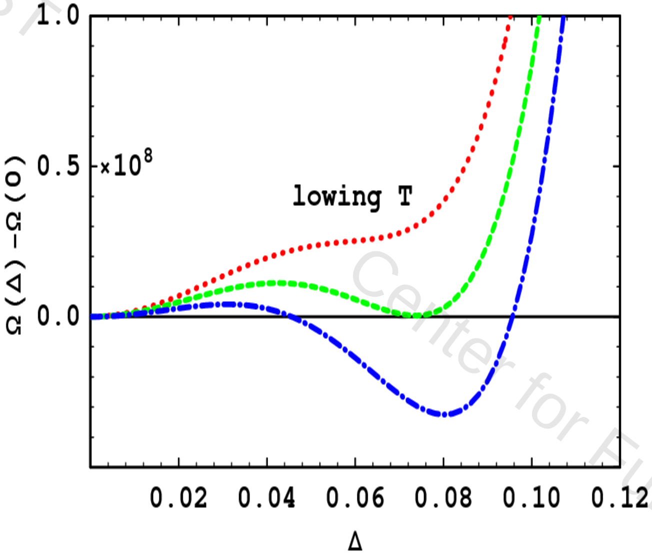
$$\Gamma_2[\bar{\mathcal{D}},\bar{\mathcal{S}}]=-\frac{1}{2}\mathrm{Tr}\{\bar{\mathcal{D}}\,\Pi[\bar{\mathcal{S}}]\}$$

D. Rischke Prog. Part. Nucl. Phys. 52 197 (2004)

Gauge field fluc. induce 1st order PT of CSC in dense QCD

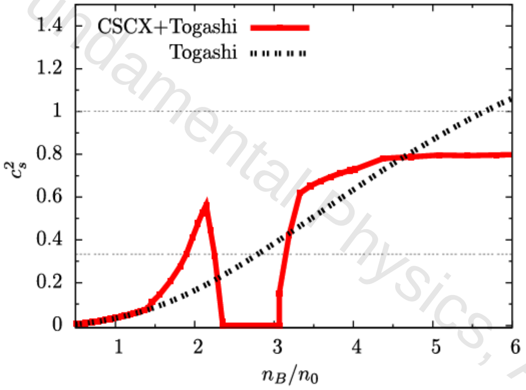
Ginnakis, Hou, Ren, Rischke, PRL 93 (04) ; PRD73 (06)

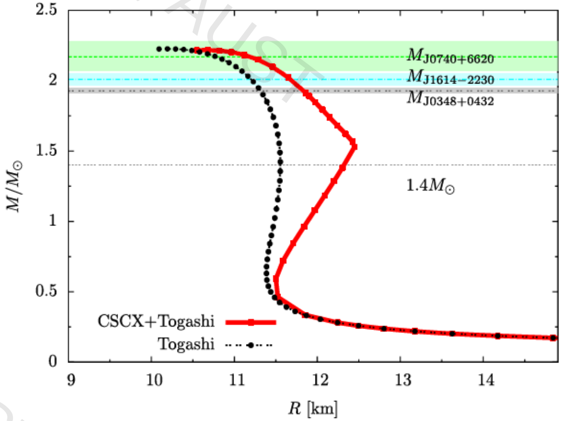




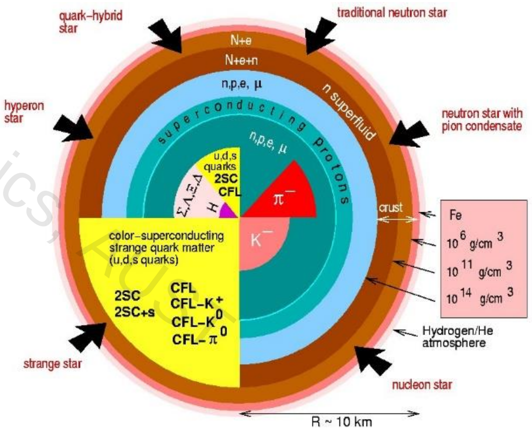
Introduction of Δ^3 term in free energy by flucts. Inducing 1st order PT in stead of 2nd order PT in MFA

QCD物质的状态方程与中子星结构



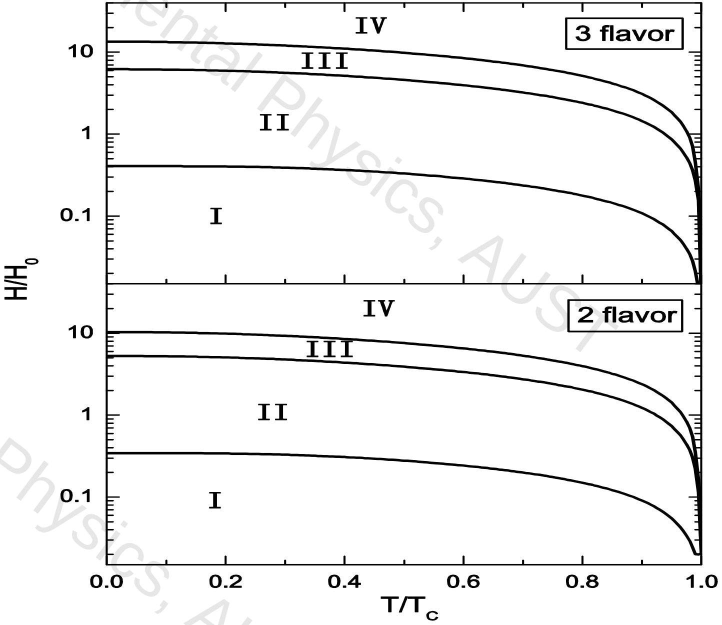


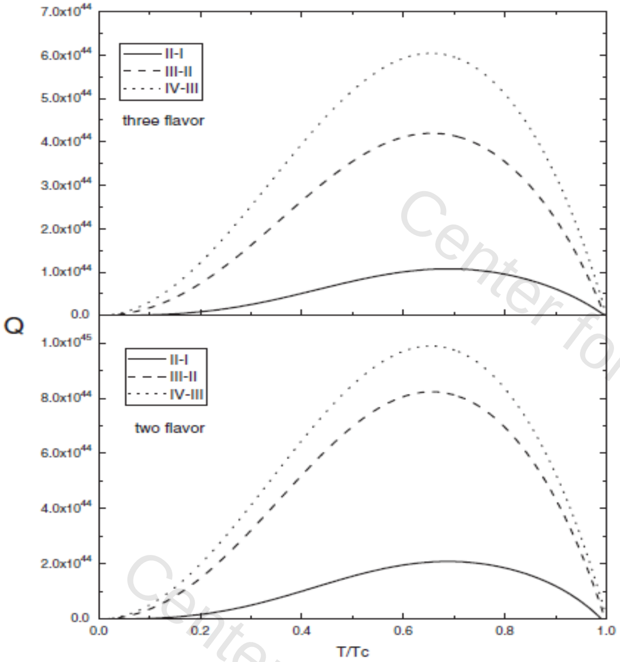
T. Kojo， DF Hou, etc.
PRD 104 (2021) 063036



Nonspherical states in dense QCD with B

	I	II	III	IV	$T_C(10^{-1}$ MeV)
Two-flavor	CSL_u, CSL_d	$(\mathrm{polar})_u$, $(\mathrm{planar})_d$	$(\mathrm{normal})_u$, $(\mathrm{polar})_d$	$(\mathrm{normal})_u$, $(\mathrm{normal})_d$	1.35
Three-flavor	CSL_u, $\mathrm{CSL}_{d,s}$	$(\mathrm{polar})_u$, $(\mathrm{planar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{polar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{normal})_{d,s}$	0.49





H0=5.44×10ˆ14 G, 1.97×10ˆ14 G

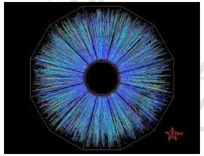
Feng, Hou, Ren,Wu, PRL 105(2010)

Wu, He, Hou, Ren, PRD84 (2011)

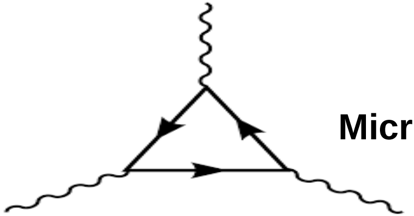
Anomalous Transports

What are their effects to the QCD transports

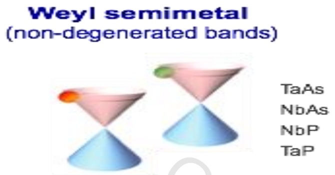
Micro-quantum anomaly + B/ Ω → macro-transport (CME/CVE)

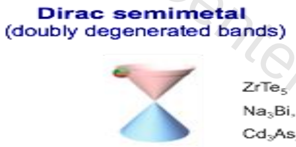


Search in HIC



Micro





B ↓ Ω

$$\boxed{\vec{J} = \sigma_5 \mu_5 \vec{B}}$$

Macro

Nature Phys.12（2016）

Science350（2015）413

Astrophysics, cosmology

Kharzeev,Liao,Voloshin,Wang: Prog.Part.Nucl.Phys.2016

The chiral vortical effect (CVE)

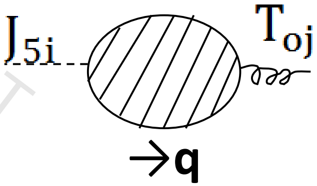
Under B & vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

$$\begin{aligned} \vec{J}_{\rm em} &= \sigma^B_{\rm em} \vec{B} + \sigma^V_{\rm em} \vec{\omega}, \\ \vec{J}_{\rm b} &= \sigma^B_{\rm b} \vec{B} + \sigma^V_{\rm b} \vec{\omega}, \\ \vec{J}_5 &= \sigma^B_5 \vec{B} + \sigma^V_5 \vec{\omega}, \end{aligned}$$

Son & Surowka

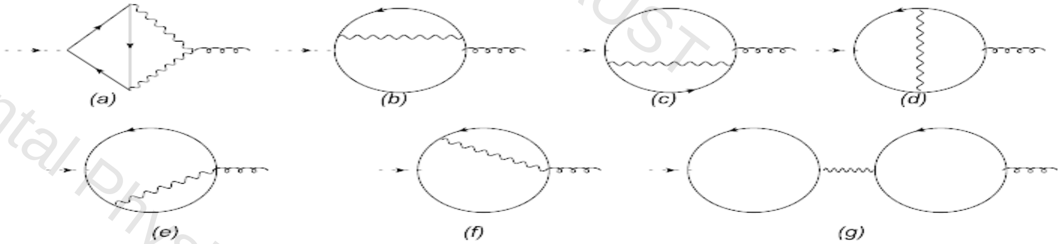
$\boldsymbol{\sigma}^{\rm B}_{\rm em}, \boldsymbol{\sigma}^{\rm B}_{\rm b}, \boldsymbol{\sigma}^{\rm B}_{5} \to$ CME $\qquad$ $\boldsymbol{\sigma}^{\rm V}_{\rm em}, \boldsymbol{\sigma}^{\rm V}_{\rm b}, \boldsymbol{\sigma}^{\rm V}_{5} \to$ CVE

Kubo formula：



$$\xrightarrow[0]{\vec{q}} \sigma^{\rm V}_5 \varepsilon_{ijk} q_k$$

Hou,Liu,Ren, PRD86 (2012) 121703®



$$\xi_5 = \frac{\mu_5^2}{2\pi^2} + cT^2$$

Are there any corrections from higher orders ?

S. Golkar and D. T. Son, arXiv:1207.5806 : No (Yes)

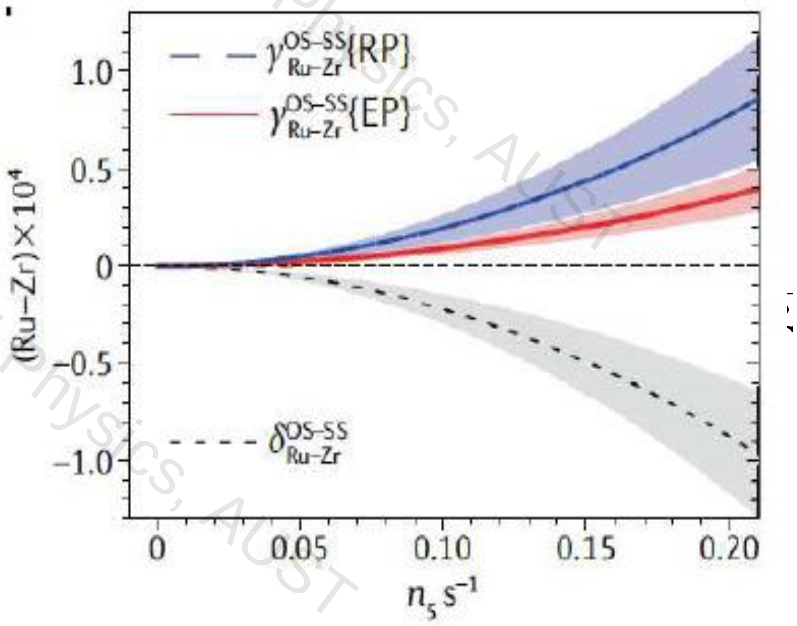
$$c = \frac{1}{12} + \frac{N_c^2 - 1}{2N_c}\frac{g_0^2}{48\pi^2} \xrightarrow[N_c \to \infty]{} \frac{1}{12} + \frac{\lambda}{96\pi^2} \qquad c = \frac{1}{12} + \frac{e_0^2}{48\pi^2}$$

反常流体力学及CME的信号的定量预测

PHYSICAL REVIEW LETTERS 125, 242301 (2020)

Signatures of Chiral Magnetic Effect in the Collisions of Isobars

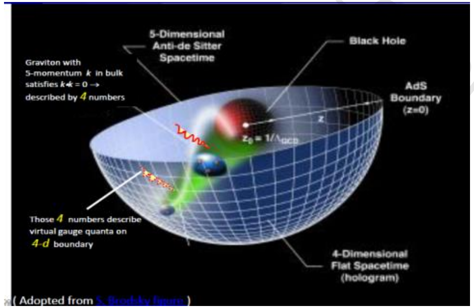
Shuzhe Shi,[1] Hui Zhang,[2,3,4] Defu Hou,[2,*] and Jinfeng Liao[5,†]



$$\zeta^{RP}_{isobar} \equiv \frac{\gamma^{OS-SS}_{Ru-Zr}\Big|_{RP}}{\delta^{OS-SS}_{Ru-Zr}\Big|_{RP}} \simeq -(0.90 \pm 0.45)$$

$$\zeta^{EP}_{isobar} \equiv \frac{\gamma^{OS-SS}_{Ru-Zr}\Big|_{EP}}{\delta^{OS-SS}_{Ru-Zr}\Big|_{EP}} \simeq -(0.41 \pm 0.27)$$

sQGP matter via AdS/CFT



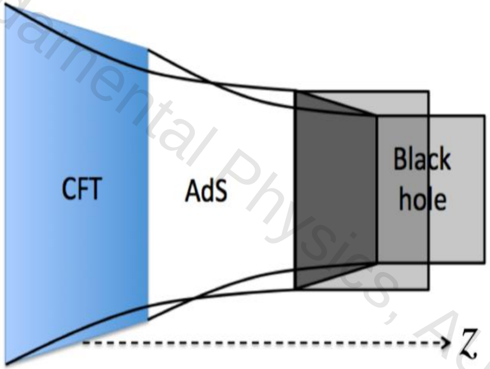
(Adopted from S. Brodsky figure)

4dim. Large-Nc strongly coupled SU(Nc) N=4 SYM (finite T).

Maldacena '97 ↕ conjecture

Witten '98

Type II B Super String theory on AdS5-BH×S5



Some complicated Field theory calculations become simple "geometric' problems in higher dimensions

Phase diagram @2+1 flavor

- **Holographic model:**

[1] R.G. Cai, etc , Phys.Rev.D 106 (2022) 12, L121902 • e-Print: 2201.02004

  - **The action:**

$$S_M = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right],$$

where the potential and kinetic functions read

$$V(\phi) = -12\cosh\left[c_1\phi\right] + \left(6c_1^2 - \frac{3}{2}\right)\phi^2 + c_2\phi^6,$$

Capturing the behavior of EOS at zero chemical potential.

$$Z(\phi) = \frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi}.$$

Capturing the flavor dynamic.

  - **The metric:**

$$ds^2 = -e^{-\eta(r)}f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx_1^2 + dx_2^2 + dx_3^2),$$

$$\phi = \phi(r), \qquad A_t = A_t(r),$$

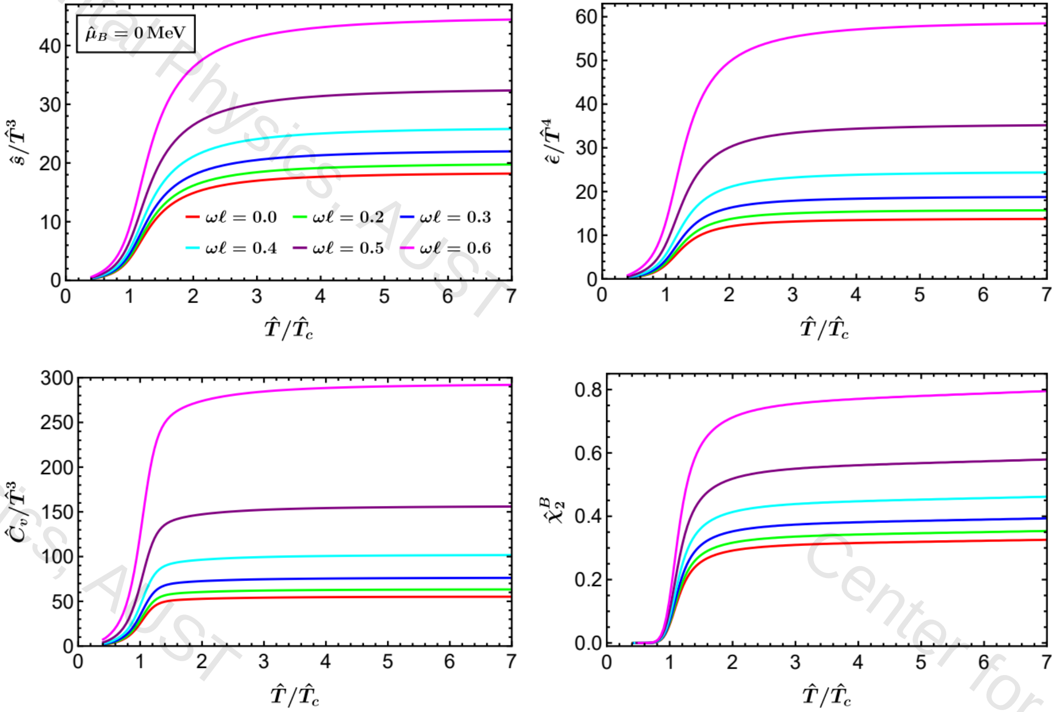
  - **The Hawking temperature and entropy density:**

$$T = \frac{1}{4\pi}f'(r_h)e^{-\eta(r_h)/2} \qquad s = \frac{2\pi}{\kappa_N^2}r_h^3.$$

effective Newton constant

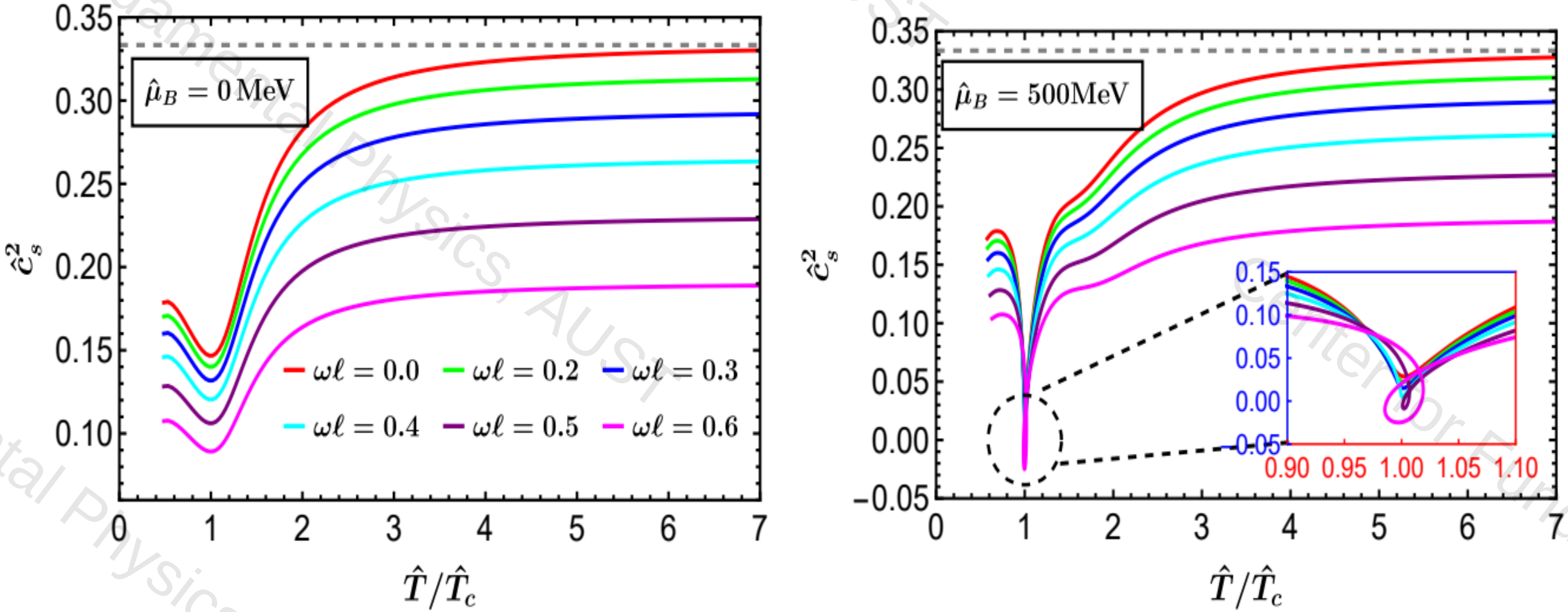
Phase diagram @2+1 flavor

- **Thermodynamics *with rotation*:**



Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, JHEP 04 (2023) 115 • e-Print: 2212.14662

➤ **Thermodynamics *with rotation*:**



Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, JHEP 04 (2023) 115 • e-Print: 2212.14662

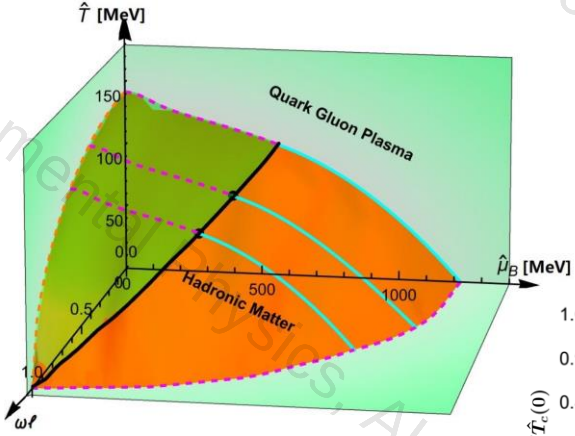
ID: Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, JHEP 04 (2023) 115 • e-Print: 2212.14662

➢ 2+1 flavor:

➢ Pure gluon($\hat{\mu}_B = 0$):

- ✓ **black solid line:** denoting the location of CEP.
- ✓ At high $\hat{T}$ and small $\hat{\mu}_B$: **Being the smooth crossover.**
- ✓ At low $\hat{T}$ and large $\hat{\mu}_B$: **Being 1st-order transition.**
- ✓ $\omega \uparrow \Longrightarrow \hat{T}_c \downarrow,\ \hat{\mu}_c \downarrow, \hat{T}_{cep} \downarrow,\ \hat{\mu}_{cep} \downarrow.$

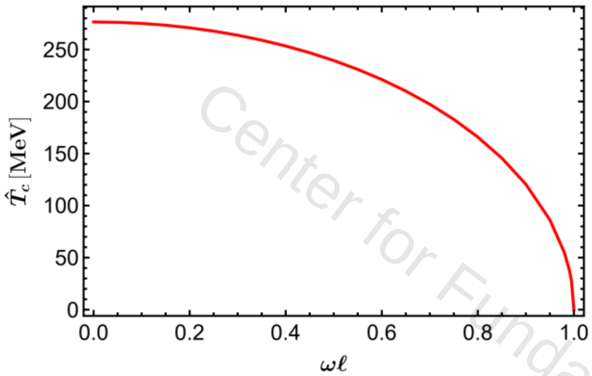
Analytically: $\hat{T}_c(\omega) = T_c\sqrt{1-\omega^2\ell^2}$

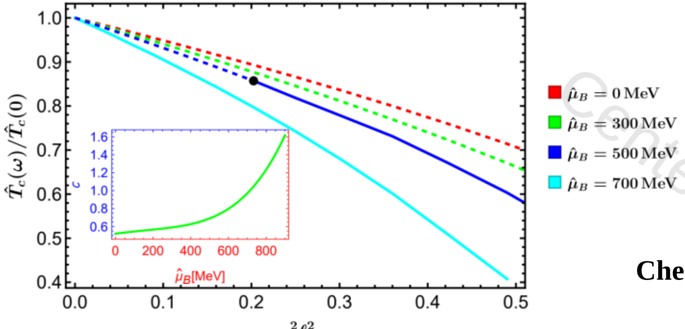


$$\hat{T}_c(\omega)/\hat{T}_c(0) \approx 1 - c\,\omega^2$$

- ✓ At finite $\hat{\mu}_B$ **and smaller** ω.

The value of c depends on $\hat{\mu}_B$.

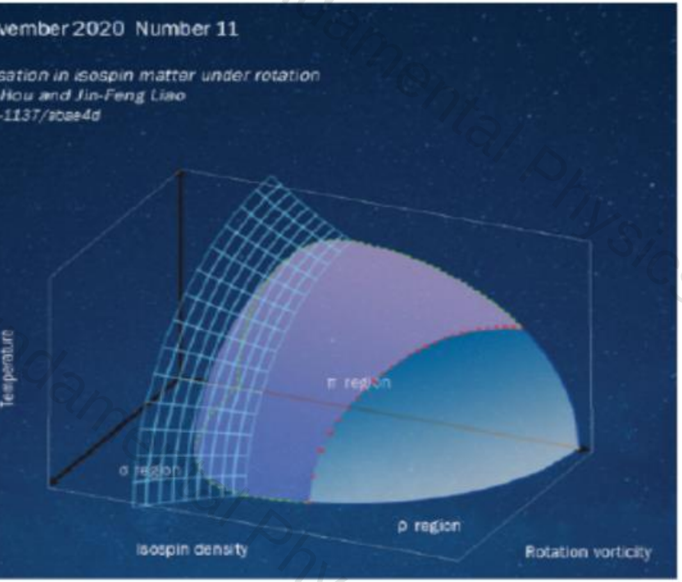


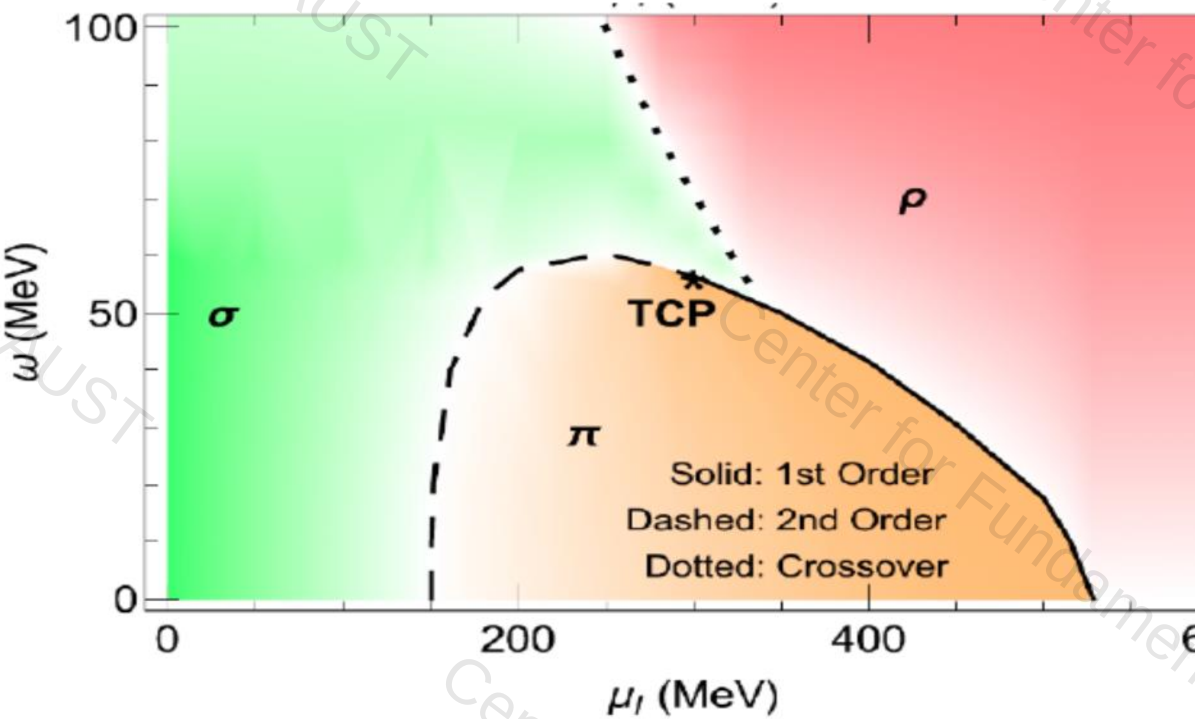


Chen, Zhang, Li, Hou , M.Huang ,JHEP 07 (2021) 132

Static isospin matter: pion superfluidity;

sospin matter under rotation: emergence of rho condensat



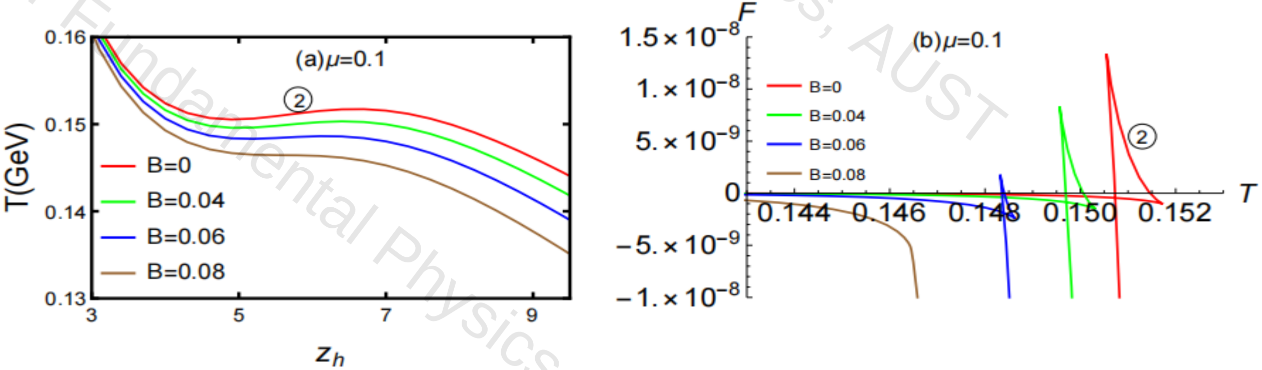


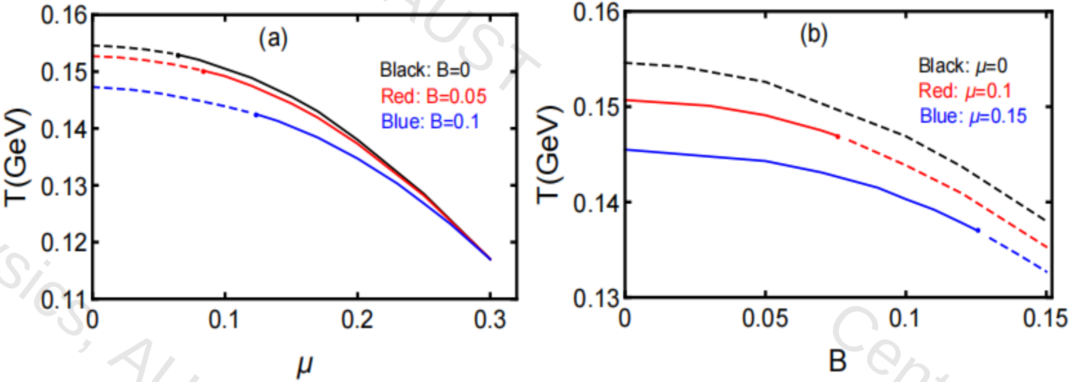
Rich phase structures found;

Could be relevant to low energy

or neutron star matter

Phase Structure with magnetic field



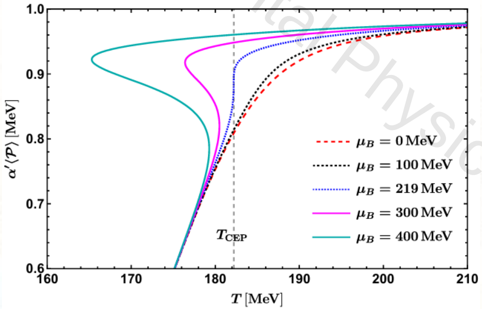


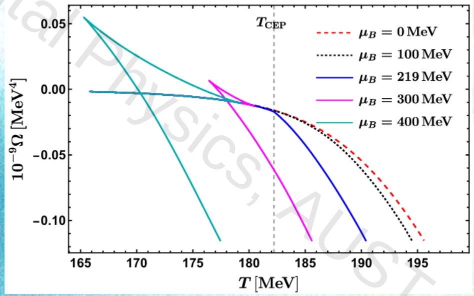
ATHIC 2008, Tsukuba, Japan, Oct. 13 - 15, 2008

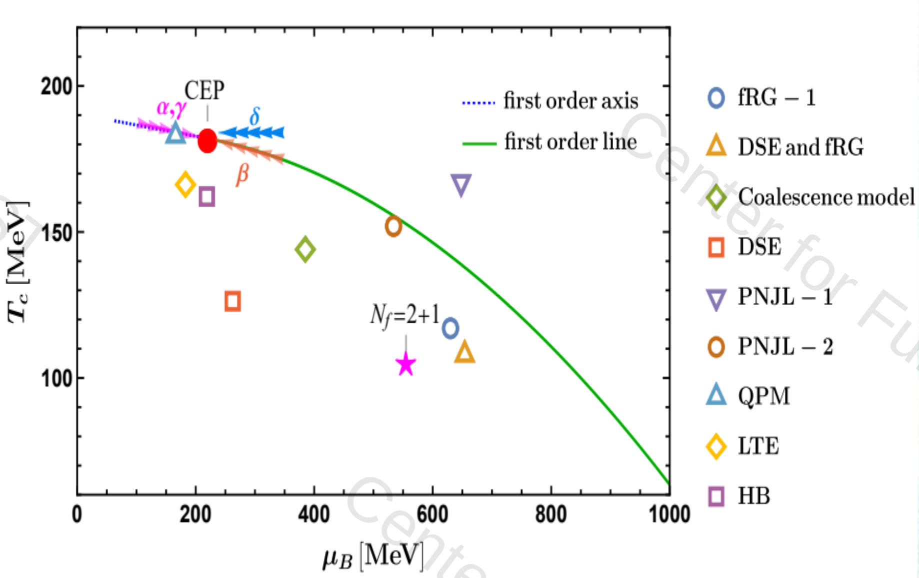
Zhou-Run Zhu, De-fu Hou, Inverse magnetic catalysis and energy loss in holographic QCD model, (arXiv:2305.12375).

Phase structure and critical phenomena in 2-flavor QCD

Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li （ Phys.Rev.D 109 (2024) 8, 086015 ）







2024/5/24

Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li （ Phys.Rev.D 109 (2024) 8, 086015 ）

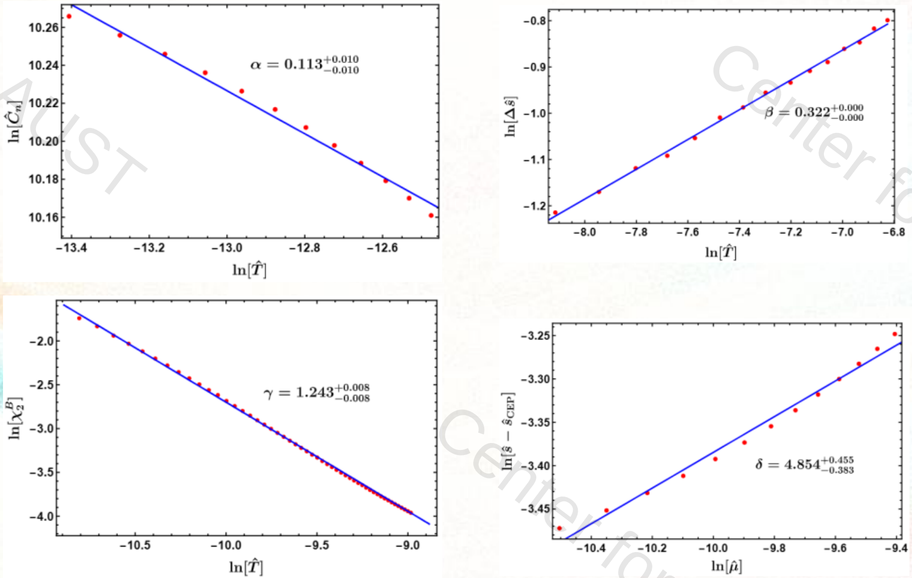
临界现象:

$$C_n \equiv T\left(\frac{\partial s}{\partial T}\right)_{n_B} \sim |T - T_{\mathrm{CEP}}|^{-\alpha}$$

$$\Delta s = s_> - s_< \sim (T_{\mathrm{CEP}} - T)^{\beta}.$$

$$\chi_2^B = \frac{1}{T^2}\left(\frac{\partial n_B}{\partial \mu_B}\right)_T \sim |T - T_{\mathrm{CEP}}|^{-\gamma}.$$

$$s - s_{\mathrm{CEP}} \sim |\mu_B - \mu_{\mathrm{CEP}}|^{1/\delta}$$

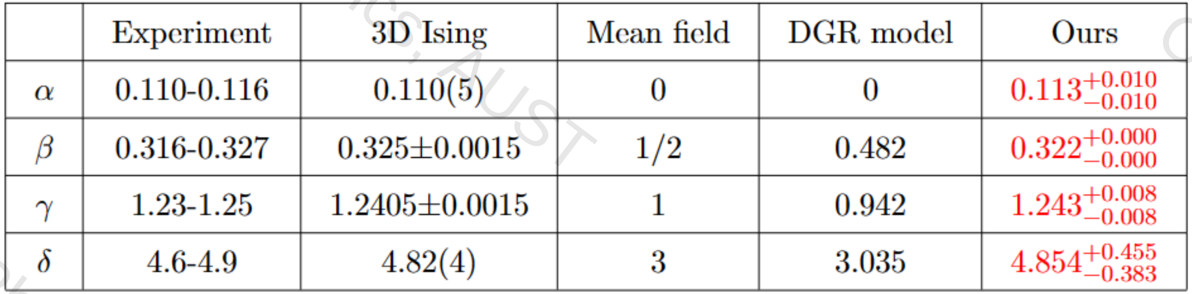


Phase structure and critical phenomena in two-flavor QCD by holography （Phys.Rev.D 109 (2024) 8, 086015）

Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li

标度关系:

$$\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(1+\delta) = 2\,.$$



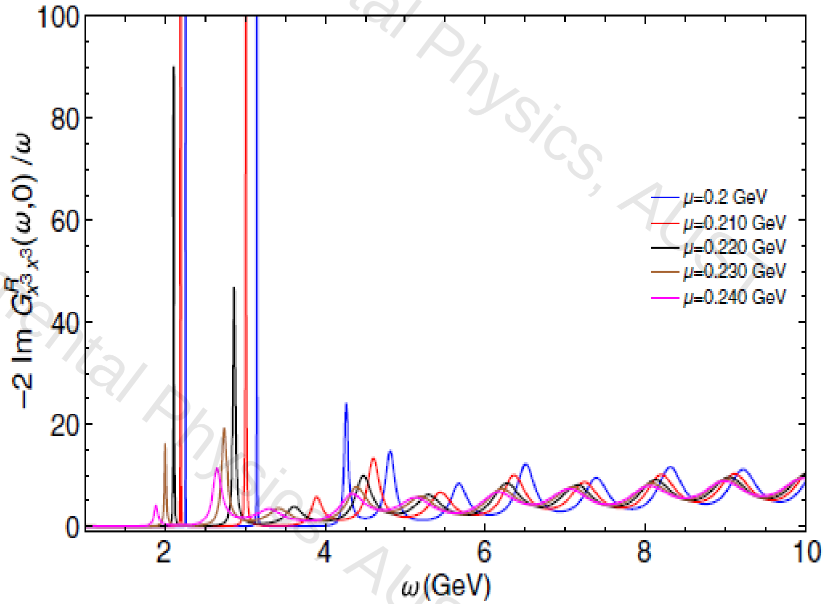
	Experiment	3D Ising	Mean field	DGR model	Ours
α	0.110-0.116	0.110(5)	0	0	$0.113^{+0.010}_{-0.010}$
β	0.316-0.327	0.325±0.0015	1/2	0.482	$0.322^{+0.000}_{-0.000}$
γ	1.23-1.25	1.2405±0.0015	1	0.942	$1.243^{+0.008}_{-0.008}$
δ	4.6-4.9	4.82(4)	3	3.035	$4.854^{+0.455}_{-0.383}$

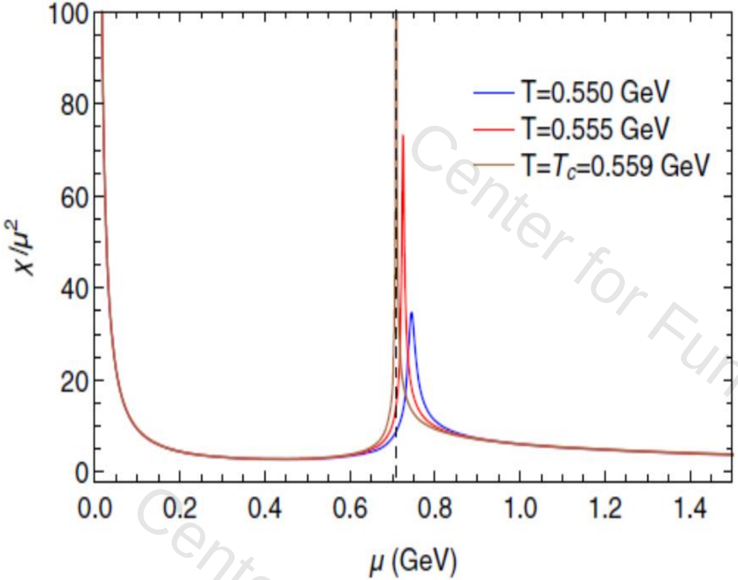
[3]O. DeWolfe, S.S. Gubser and C. Rosen, A holographic critical point, Phys. Rev. D 83 (2011) 086005 [1012.1864].

[4]N. Goldenfeld, Lectures on phase transitions and the renormalization group (1992).

The spectral function of heavy vector mesons

Mamani , Hou, Braga, PRD 105, 126020 (2022)



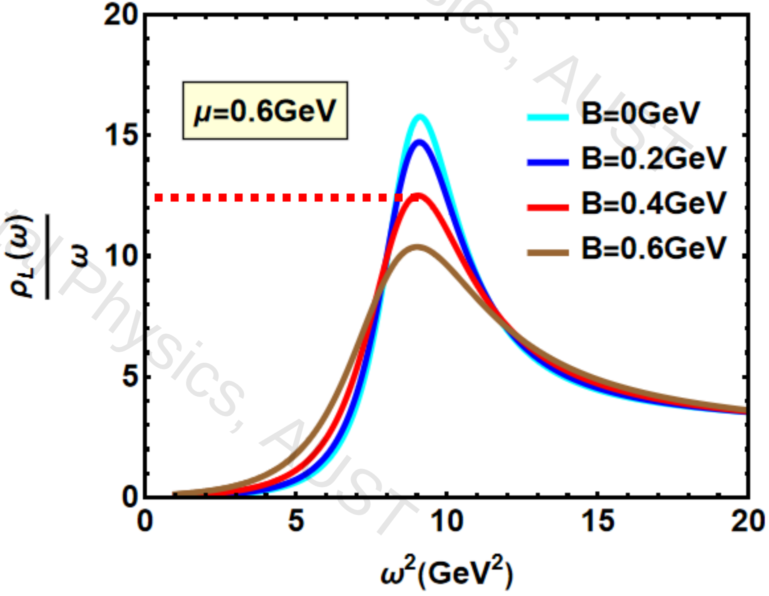


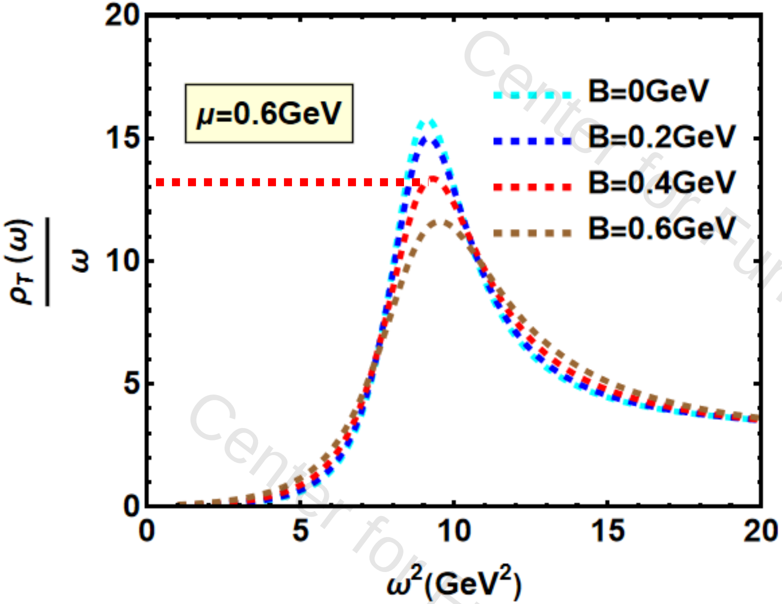
J/Ψ

- **Spectral function:**

Yan-Qing Zhao, Defu Hou, Eur.Phys.J.C 82 (2022) 12, 1102 • e-Print: 2108.08479

As increasing magnetic field, the dissociation effect increases and it is stronger for the parallel case.





J/Ψ 和 Υ(1S) 的谱函数

W.B. Chang and De-Fu Hou, Phys. Rev. D 109, 086010 (2024).

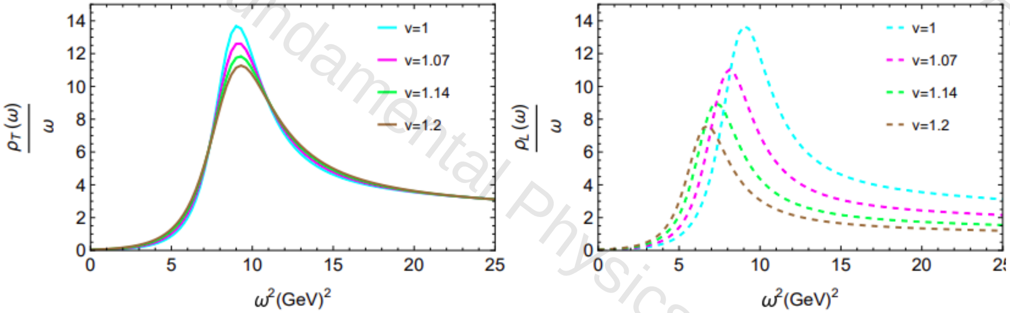


图4. 不同的各向异性参数下，J/Ψ 的谱函数

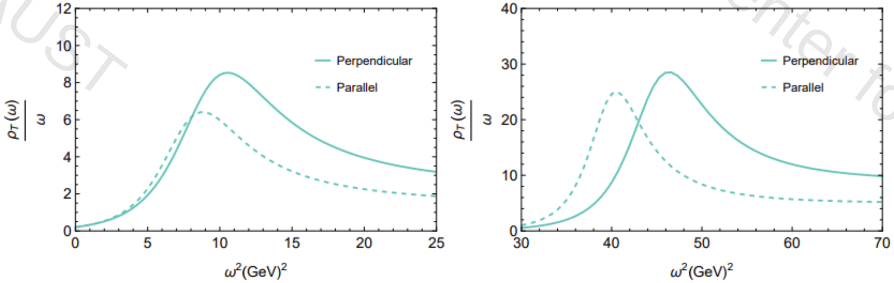


图6. 实线代表各向异性与极化方向垂直
虚线代表各向异性与极化方向平行

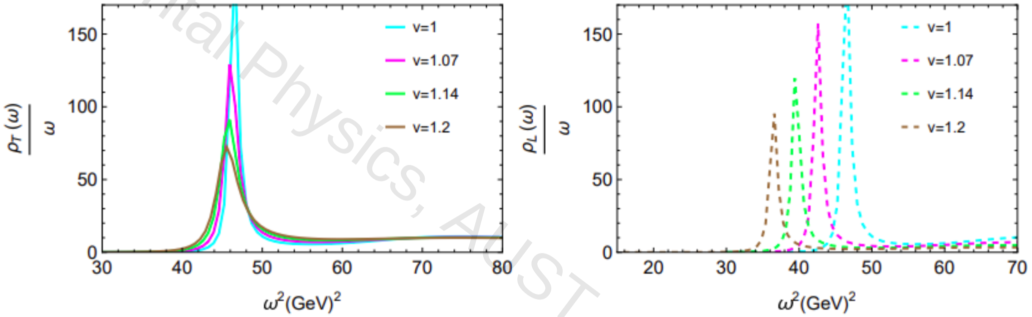


图5. 不同的各向异性参数下，Υ(1S) 的谱函数

1） 各向异性会加速束缚态的溶解
2） 各向异性方向与极化方向平行时，这一其效应更为显著

J/Ψ 和 Υ(1S) 的谱函数

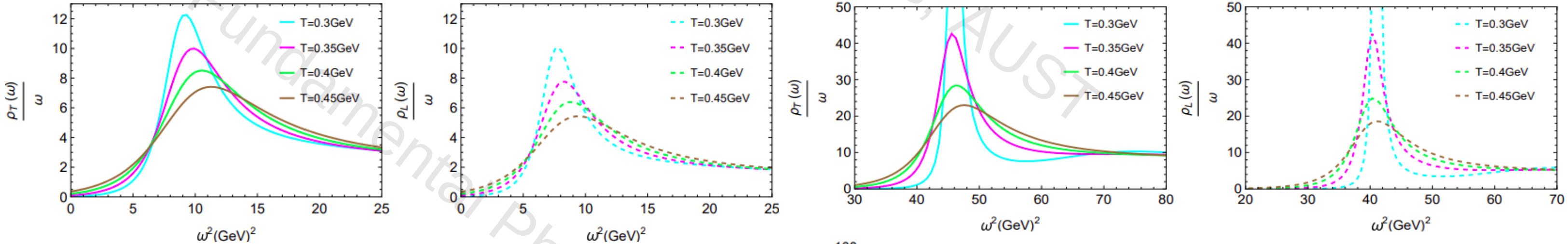


图7. 温度

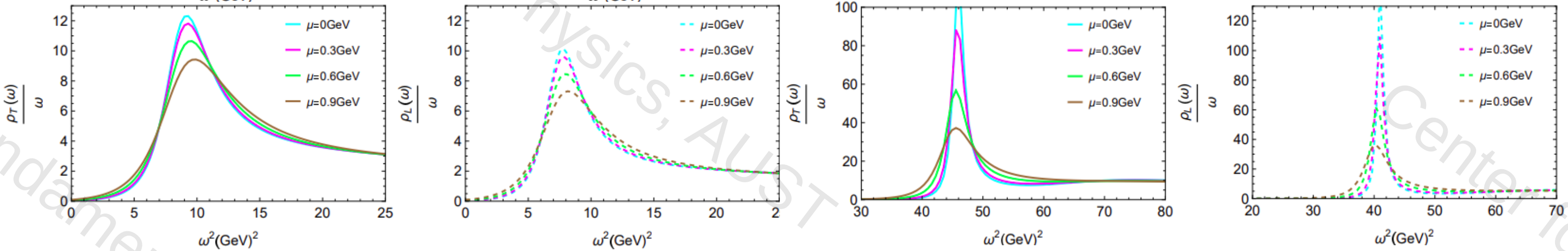


图8. 化学势

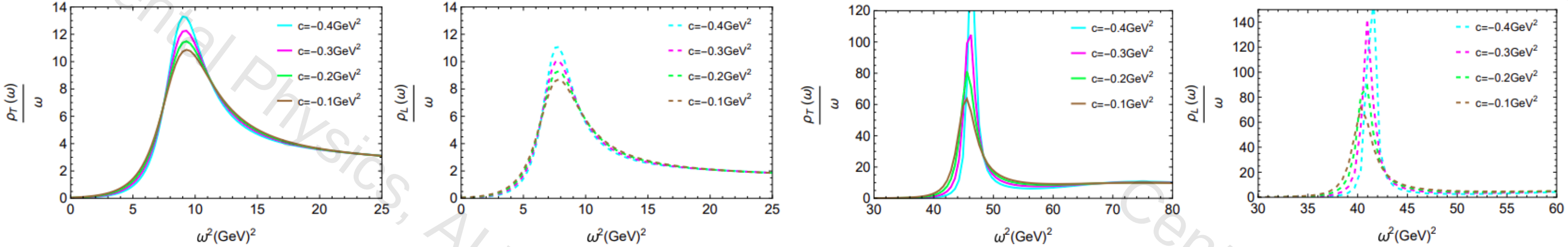
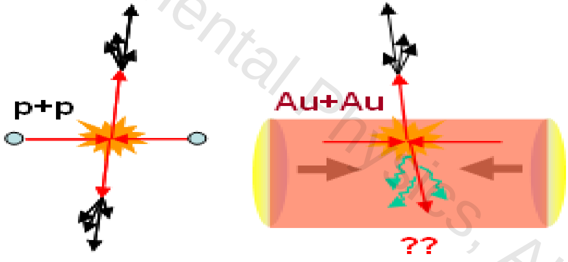


图9. 弯曲因子

3) 有限温度和密度效应会导致重夸克偶素的熔解
4) 更大的弯曲因子会增强溶解效应

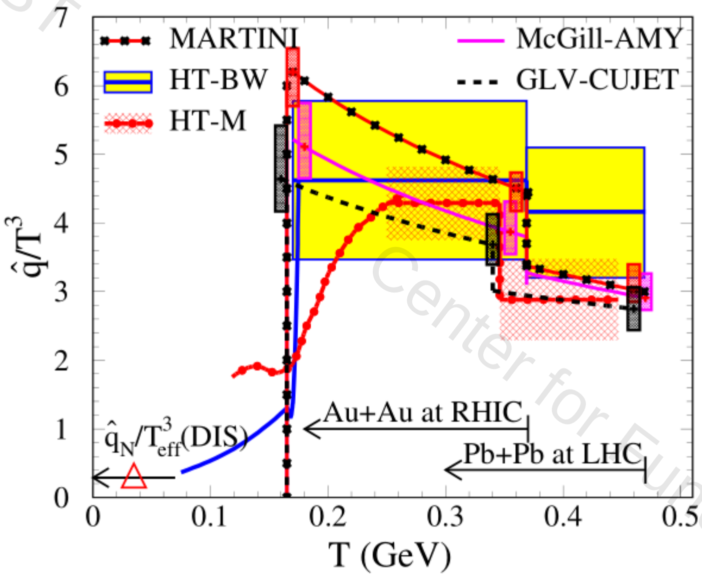
W.B. Chang and DH, Phys. Rev. D 109, 086010 (2024).

Jet quenching in QGP



$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller,Peigne, Schiff (1996):

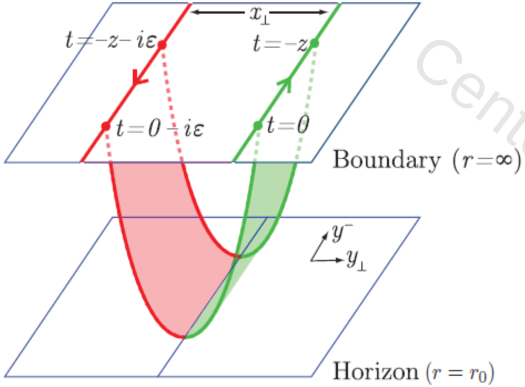


Jet Collaboration, PRC 90,014909(2014)

Energy loss and jet quenching

$$\langle W^A[C]\rangle \approx exp(-\frac{1}{4\sqrt{2}}\hat{q}L_-L^2) \quad \langle W^F[C]\rangle \approx exp[-S_I]$$

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$



H. Liu, K. Rajagopal, and U. A. Wiedemann,
Phys. Rev. Lett. 97, 182301 (2006).

NL correction to jet quenching parameter

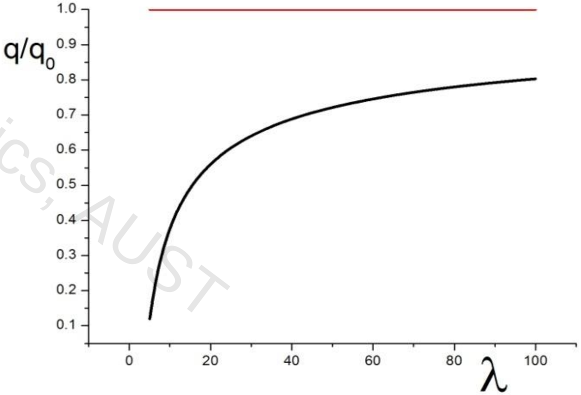
Zhang, Hou, Ren, JHEP1301 (2013) 032

dominant

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^{3}[1-\underline{1.97\lambda^{-1/2}}+O(\lambda^{-1})]$$

$$1 - \underline{1.765\lambda^{-3/2}}.$$

Armesto et al JHEP09 (06)



相变温度附近的喷注淬火参数

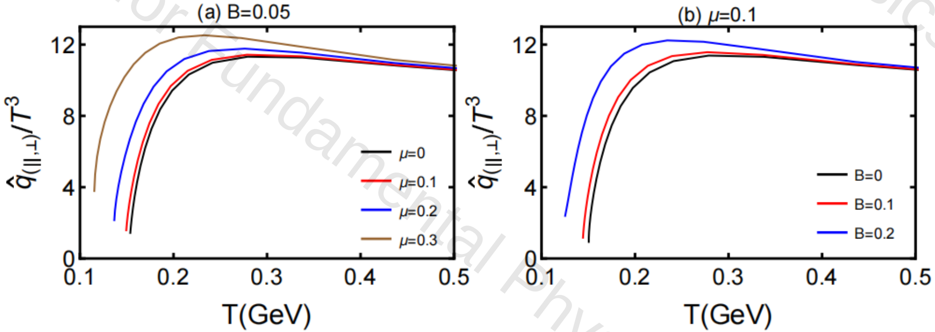


图10. $\hat{q}_{(\parallel,\perp)}$

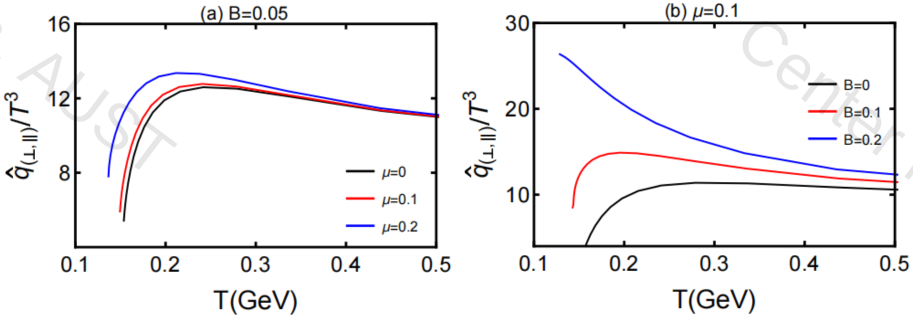


图11. $\hat{q}_{(\perp,\parallel)}$

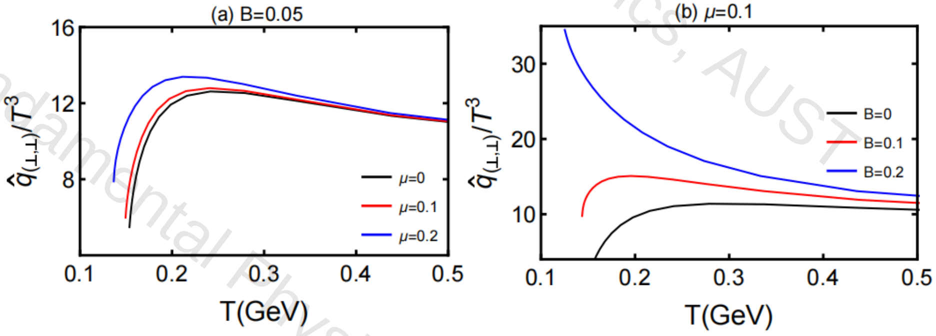


图12. $\hat{q}_{(\perp,\perp)}$

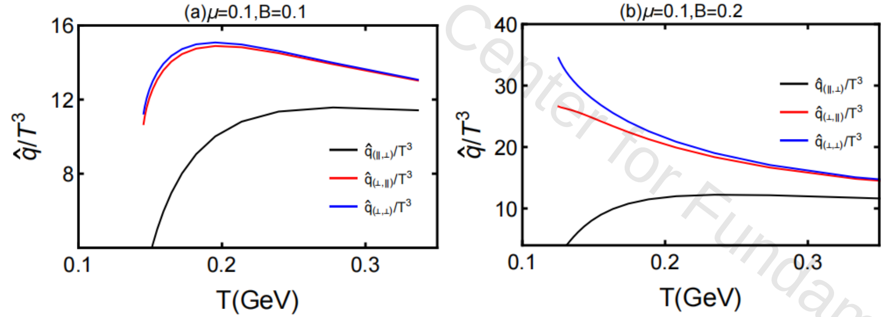


图13. $\hat{q}/T^3$

图10-12：$\hat{q}$在T_c附近有增强，与格点(150-250MeV)相符合*；
峰值所对应温度随μ/B增加而降低；μ、B增强了$\hat{q}$

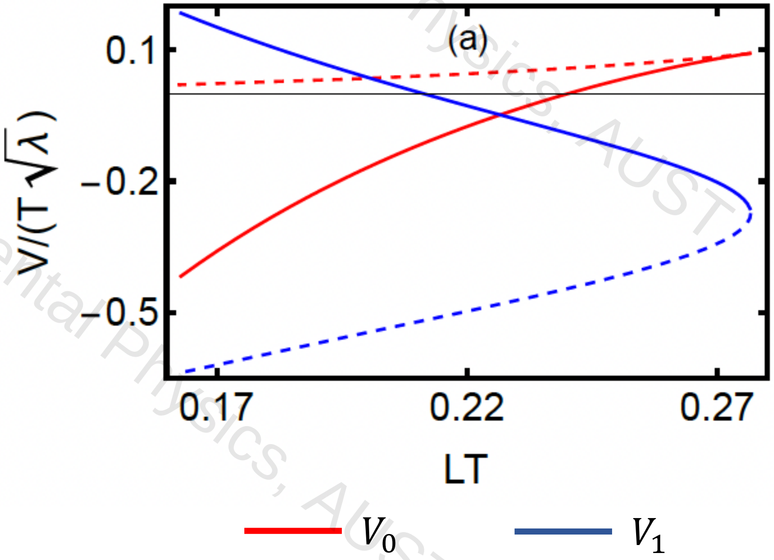
图13： $\hat{q}_{(\perp,\perp)} > \hat{q}_{(\perp,\parallel)} > \hat{q}_{(\parallel,\perp)} \rightarrow \hat{q}_{\perp} > \hat{q}_{\parallel}$

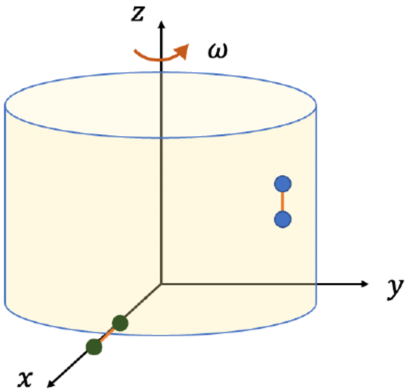
Amit Kumar,Abhijit Majumder,and Johannes Heinrich Weber. PRD 106 (2022) 3, 034505

Zhou-Run Zhu, De-fu Hou, Inverse magnetic catalysis and energy loss in holographic QCD model, (arXiv:2305.12375).

Heavy quark potential $V_{\parallel}$

$$V(L) = V_0(L) + \omega^2 l_0^2 V_1(L) + O(\omega^4)$$





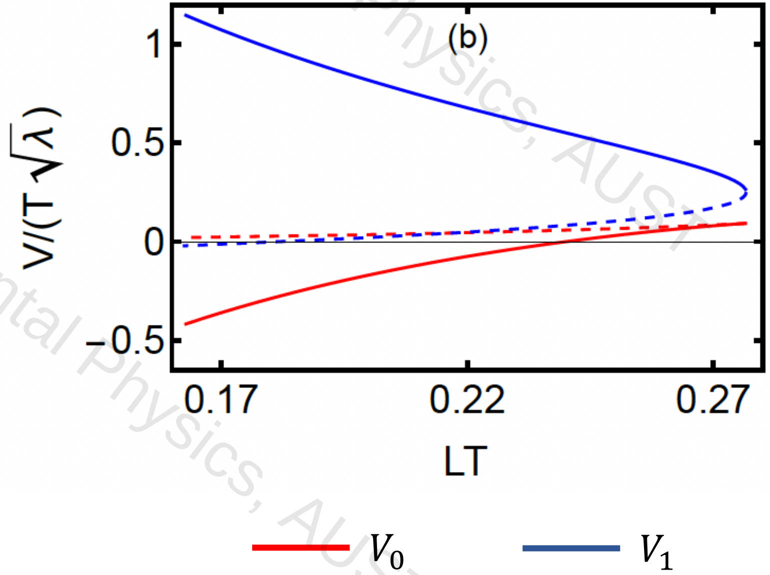
1. Binding force ↓
2. Force range ↑

$$\delta L = \frac{|V_1(L_0)|}{V_0'(L_0)} \omega^2 l_0^2 \qquad V_0(L_0) = 0$$

J.X. Chen, DH, H.C Ren , JHEP03 (2024) 171 , arXiv: 2308.08126

Heavy quark potential $V_{\perp}$

$$V(L) = V_0(L) + \frac{1}{4}\omega^2 L^2 V_1(L) + O(\omega^4)$$



1. Potential ↑

2. Force range ↓

$$V_0(L_0) = 0$$

$$\delta L = -\frac{V_1(L_0)}{4V_0'(L_0)}\omega^2 L_0^2$$

Dragging force

J.X. Chen, DH, H.C Ren , JHEP03 (2024) 171 , arXiv: 2308.08126

Components of drag force

$$\frac{dp_\mu}{dt} = -\frac{1}{2\pi\alpha'}\frac{\partial\mathcal{L}}{\partial\left(\frac{\partial X^\mu}{\partial r}\right)} \longrightarrow$$

$$\downarrow$$

Drag force of μ component

Azimuthal $$\frac{dp_\phi}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\omega l_0^2\frac{1}{\sqrt{1-v^2}}$$

Radial $$\frac{dp_l}{dt} = -\omega^2 l_0\frac{1}{\sqrt{1-v^2}}\left[\frac{T\sqrt{\lambda}}{2(1-v^2)^{\frac{1}{4}}} - \boxed{m_{rest}}\right] \longrightarrow$$

cut off $r_m = 2\pi\alpha' m_{rest}$

Longitudinal $$\frac{dp_z}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}\left(1+\underline{\frac{\omega^2 l_0^2}{2}\frac{1}{1-v^2}}\right)$$

$\propto \omega^2$

$$\boxed{\omega = 0} \Longrightarrow \frac{dp_\phi}{dt} = 0, \quad \frac{dp_l}{dt} = 0, \quad \frac{dp_z}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}$$

S. S. Gubser, PRD74, 126005 (2006); C. P. Herzog, etc. JHEP 07, 013 (2006).

Summary outlook

Properties of strongly interacting matter under extreme conditions are very interesting!

- **QCD Phase diagram under magnetic field & rotation (IMC)**
- **Heavy quark potential and dragging force**
- **Spectral function and heavy meson melting**
- **How to understand the different results of rotation from lattice？**

  （Polarization induced by Magnetic field and rotation?

Summary

Properties of strongly interacting matter under extreme conditions are very interesting!

- **QCD Phase diagram under rotation and Magnetic field**
- **Transport properties of rotating magnetized matter**

Heavy quarkonia
Jet quenching and energy loss

Thank you very much for your attention!

相变温度附近的重夸克能损

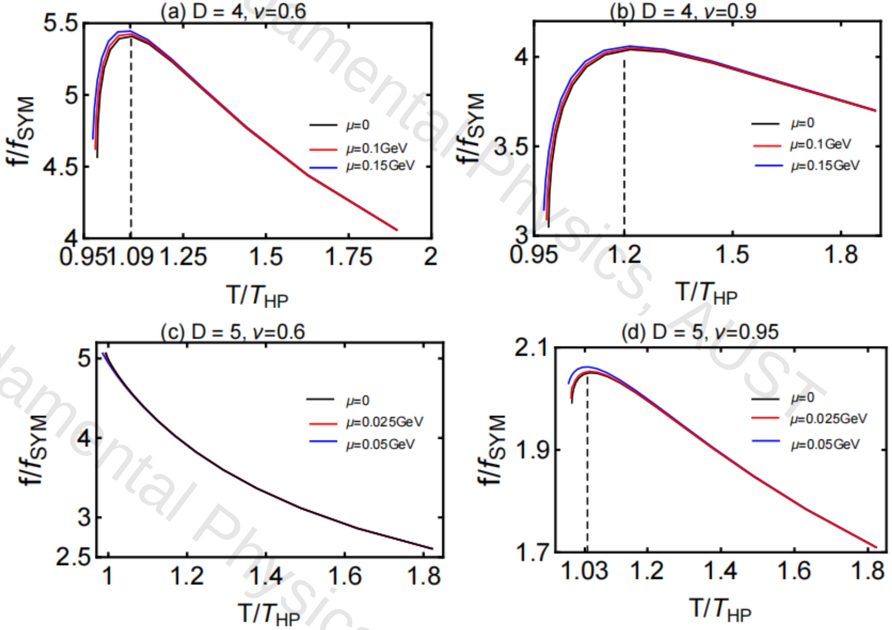


图24. 化学势的对能损的影响

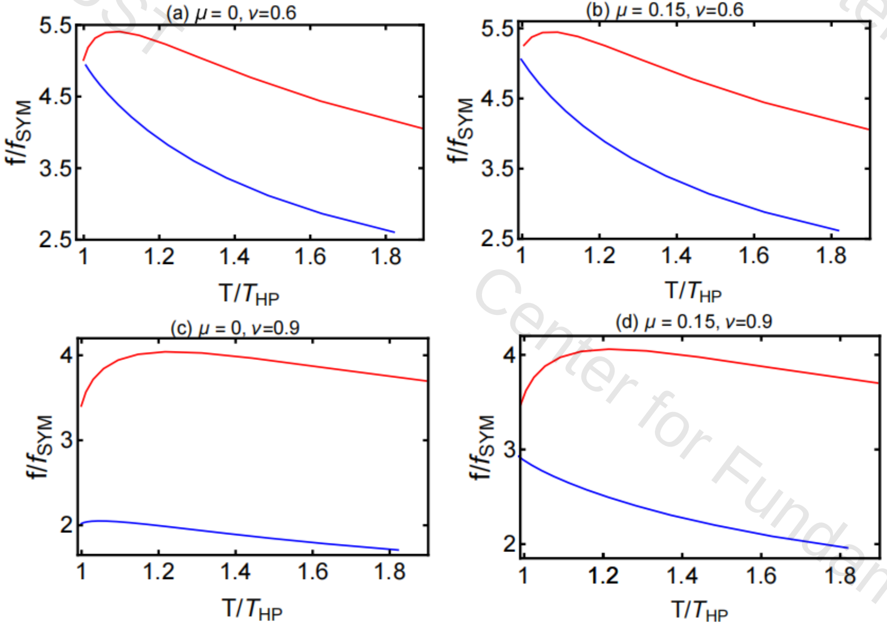


图25. 不同维度下的能损

1) 在T_c附近有增强，峰值所对应温度随μ增加而降低；
2)在较低维度时，重夸克损失更多能量。

Zhou-Run Zhu, Jun-Xia Chen, Xian-Ming Liu, De-fu Hou, Eur.Phys.J.C 82 (2022) 6,560.

De Hass-van Alphen Effect with Rotation

Shu-Yun Yang, Ren-Da Dong, DF Hou, Hai-Cang Ren, PRD 107, 076020 (2023)

finite T

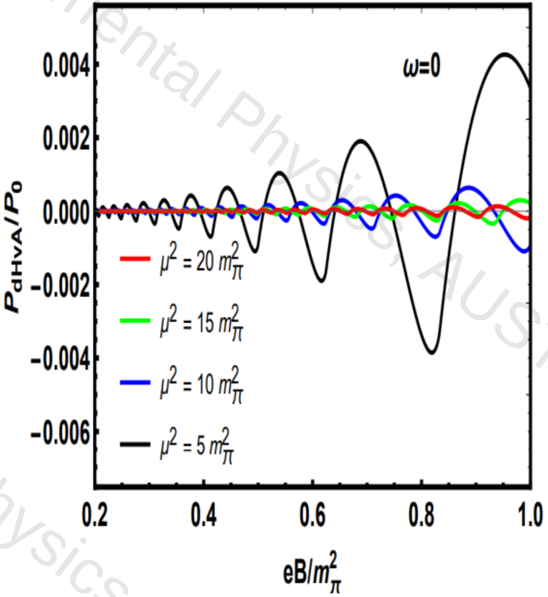
$$P_{\mathrm{dHvA}} = -\frac{(eB)^{\frac{1}{2}}}{2\pi^2 R^2}\sum_{l=1}^{\infty}\frac{1}{l^{3/2}}\sum_{M>0}\frac{\cos\left[\frac{l\pi}{eB}(\mu+M\omega)^2-\frac{\pi}{4}\right]}{\sinh\frac{2l\pi^2 T(\mu+M\omega)}{eB}}.$$

Zero T:

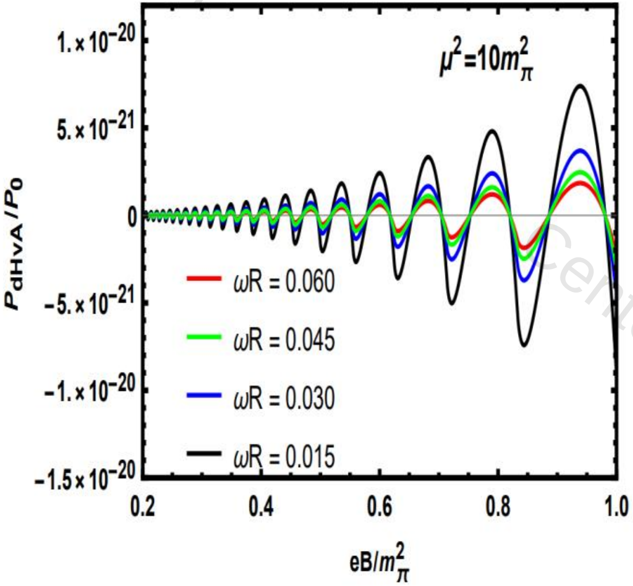
$$P_{\mathrm{dHvA}} = -\frac{(eB)^{\frac{3}{2}}}{4\pi^4 R^2}\sum_{M>0}\frac{1}{\mu+M\omega}\sum_{l=1}^{\infty}\frac{1}{l^{5/2}}\cos\left[\frac{l\pi}{eB}(\mu+M\omega)^2-\frac{\pi}{4}\right]$$

In rotation, the thermodyn Equl. is established under a AM. The equal distrib. of different AM states within a LL is offset by the nonzero AV with higher AM more favored than lower ones, which amounts to lifting the degeneracy of LL. The dHvA oscillation is thereby expected to be reduced by rotation

De Hass-van Alphen Effect with Rotation



dHvA in NS with rotation (R=1km)

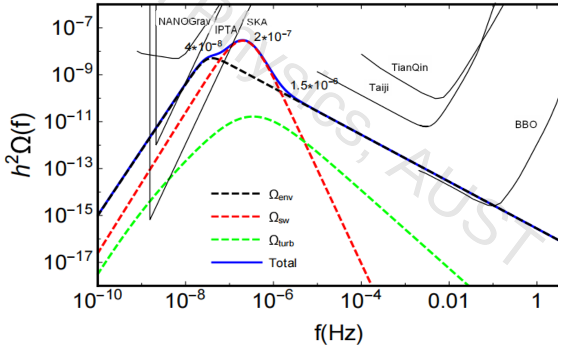


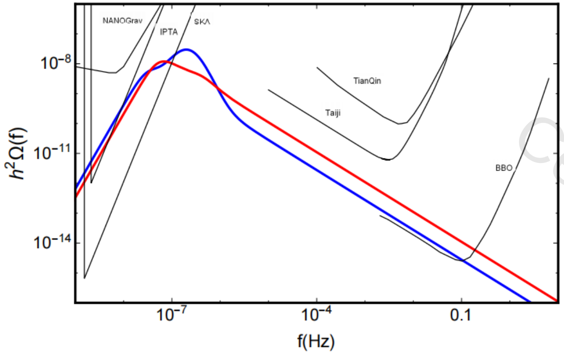
Huge suppression of dHvA oscill.(17 order)
due to large size

Shu-Yun Yang, Ren-Da Dong, DF Hou, Hai-Cang Ren, PRD 107, 076020 (2023)

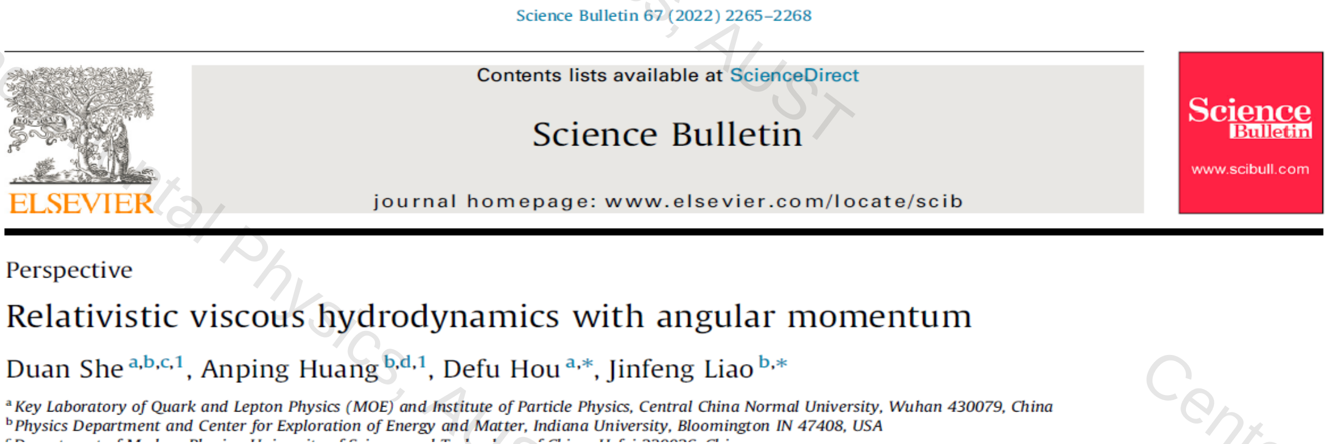
Gravitational waves from holographic QCD phase transition .

Zhou-Run Zhu, Jun Chen, Defu Hou. Eur.Phys.J.A 58 (2022) 6, 104





结论：峰值频率是由声波决定；胶子凝聚抑制了总引力波的能量密度和峰值频率。



Science Bulletin 67 (2022) 2265–2268

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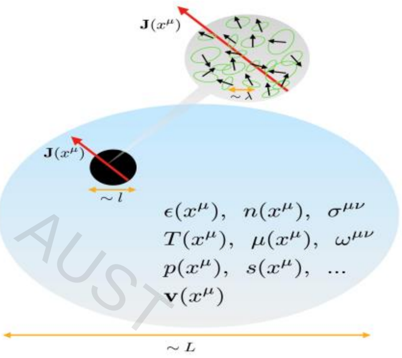
Perspective

Relativistic viscous hydrodynamics with angular momentum

Duan She [a,b,c,1], Anping Huang [b,d,1], Defu Hou [a,*], Jinfeng Liao [b,*]

[a] *Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

[b] *Physics Department and Center for Exploration of Energy and Matter, Indiana University, Bloomington IN 47408, USA*



$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu N^\mu = 0$$

$$\partial_\mu J^{\mu\alpha\beta} = 0$$

$$J^{\mu\alpha\beta} = (x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}) + \Sigma^{\mu\alpha\beta}$$

$$\partial_\mu S^\mu \geq 0$$

$$\epsilon + p = Ts + \mu n + \omega_{\mu\nu}\sigma^{\mu\nu}$$

$$d\epsilon = Tds + \mu dn + \omega_{\mu\nu} d\sigma^{\mu\nu}$$

$$dp = sdT + nd\mu + \sigma^{\mu\nu} d\omega_{\mu\nu}$$

Viscous Hydro with Ang. Mom.

Write down all allowed Lorentz structures to the correct order of gradient expansion

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + 2u^{(\mu}q^{\nu)} + \pi^{\mu\nu} \\
N^\mu &= n u^\mu \\
\Sigma^{\mu\alpha\beta} &= u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha}\Delta^{\mu\beta]}\Phi + 2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta} \\
S^\mu &= p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}
\end{aligned}
$$

Plug these into the entropy current divergence and look for conditions of positivity:

$$
\begin{aligned}
\Pi &= -\zeta\theta \\
\pi^{\mu\nu} &= 2\eta\nabla^{\langle\mu}u^{\nu\rangle} \\
q^\mu &= \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right) = -\frac{\lambda n T^2}{\epsilon+p}\left[\nabla^\mu\left(\frac{\mu}{T}\right) + \frac{\sigma^{\alpha\beta}}{n}\nabla^\mu\left(\frac{\omega_{\alpha\beta}}{T}\right)\right] \\
\Phi &= -\chi_1 u^\alpha \nabla^\beta\left(\frac{\omega_{\alpha\beta}}{T}\right) \\
\tau_{(s)}^{\mu\beta} &= -\chi_2 u^\alpha\left[\left(\Delta^{\beta\rho}\Delta^{\mu\gamma} + \Delta^{\mu\rho}\Delta^{\beta\gamma}\right) - \frac{2}{3}\Delta^{\mu\beta}\Delta^{\rho\gamma}\right]\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right) \\
\tau_{(a)}^{\mu\beta} &= -\chi_3 u^\alpha\left(\Delta^{\beta\rho}\Delta^{\mu\gamma} - \Delta^{\mu\rho}\Delta^{\beta\gamma}\right)\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right) \\
\Theta^{\mu\alpha\beta} &= -\chi_4\left(u^\beta u^\rho \Delta^{\alpha\delta} - u^\alpha u^\rho \Delta^{\beta\delta}\right)\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right) + \chi_5 \Delta^{\alpha\delta}\Delta^{\beta\rho}\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right)
\end{aligned}
$$

Five new positive angular momentum transport coefficients: